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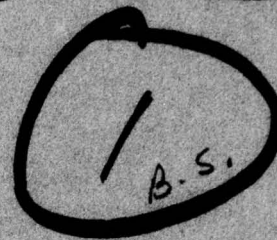
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AN ASSESSMENT OF SOME FORMS OF FOLLOWER-TOOTH REDUCTION GEAR AND ITS APPLICATION TO HELICOPTER MAIN ROTOR GEARBOXES.

by

10 T.R. Smith

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Consideration has been given to the mechanism's application to helicopter main gearboxes and a design study formulated. It is not considered competitive with current types because:

(1) Although the specific mass of a basic follower-tooth gear is about the same as that of conventional transmissions in similar configurations, the latter have more potential for mass reduction, and their efficiency is higher.
(2) The addition of tail rotor and ancillary drive take-offs would increase complexity to a greater extent than in conventional systems.
(3) Due to its novel arrangement and the type and function of the speed reducing components, the mechanism would be more sensitive to environmental ...

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T. R. Smith

SUMMARY

This Report presents a critical assessment of the cylindrical rotary form of the follower-tooth reduction gear, indicating deficiencies and suggesting improvements.

Consideration has been given to the mechanism's application to helicopter main gearboxes and a design study formulated. It is not considered competitive with current types because:

(1)   Although the specific mass of a basic follower-tooth gear is about the same as that of conventional transmissions in similar configurations, the latter have more potential for mass reduction, and their efficiency is higher.

(2)   The addition of tail rotor and ancillary drive take-offs would increase complexity and weight to a greater extent than in conventional systems.

(3)   The geometric arrangement and the type and function of the speed reducing components are such that the mechanism would be more sensitive to environmental conditions.

(4)   Although components are geometrically simple, high manufacturing standards, quality control and assembly difficulties would reflect adversely on costs.

(5)   Refurbishment would probably entail replacement of most power-transmitting components, bearings and seals, whereas conventional systems normally require only new bearings and seals.

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1   INTRODUCTION

Primary power sources, such as the gas turbine engine, operate most efficiently at high speed. For many applications the components they are required to drive, such as propellers or wheels, must rotate at much lower speeds. Consequently speed reducing mechanisms are required between the high speed engine output shaft and the final driven members.

Currently, much attention is being focussed on the gas turbine engine driven main rotor gearboxes of helicopters. Generally, the output speed from the engine is about 5000 rev/min (reduced from about 20000 rev/min via an engine mounted epicyclic or helical spur gear stage) and the main rotor gearbox provides the speed reduction from 5000 rev/min to main rotor speed (in the range 200-400 rev/min). These gearboxes use conventional gearing such as spiral bevels, epicyclies and helical spurs, and usually two or three stages of gearing are required to provide speed reduction from about 5000 rev/min to main rotor speed. Because the speeds of components within the gearbox are low the torque is extremely high for high power transmission (>500 kW), and large bearings and gears are needed to achieve sufficient load capacity and life. It follows that these gearboxes contain numerous components and pose problems of reliability, weight, volume and power losses.

Considerable effort is now being applied to alleviate these problems on current helicopters and to ensure that such problems are minimised in future designs. Attempts are being made to reduce total gearbox weight by using high strength materials, by the introduction of conformal gears, and by optimising the arrangement of conventional components. In addition, other mechanisms that might perform the same function more advantageously are being considered. The essential requirement is for a high ratio speed reducer capable of transmitting high power and torque with low power losses in a single compact stage.

Molyneux[1] has discussed in outline a variety of follower-tooth reduction gear arrangements that he claims may meet these requirements. However, a superficial analysis suffices to establish that, where high transmission efficiency is a paramount consideration, only the cylindrical-rotary-eccentric configuration has any likelihood of competing with more conventional arrangements. Attention is accordingly focused on the cylindrical-rotary-eccentric follower-tooth reduction gear and, in order to show its general industrial capabilities, the theoretical performance of a range of sizes has been estimated. More specifically, a detailed design study has been made of a rotary design with requirements corresponding to those of a typical present-day helicopter main gearbox and consideration is given

to the problems of manufacture, environmental operation and theoretical perform-
ance. In this study the input speed to the follower-tooth reduction gear is
assumed to be that following the turbine engine reduction gearbox, ie 5000 rev/min,
and the output speed is assumed to be 250 rev/min with a transmitted power of
1492 kW (2000 hp).

To make the Report reasonably compact details of many of the design calcu-
lations involved are omitted although the assumptions on which they are based are
shown in the Appendices

2 OPERATION PRINCIPLES OF THE FOLLOWER-TOOTH MECHANISM

Referring to Fig 1, if one considers two regular, even, periodic functions
identical in amplitude and form except that the wavelength of one is a constant
multiple of the wavelength of the other, the points of intersection of these
functions occur in two distinct groups. In one group X , say, the gradients of
the functions at the common point are of the opposite sign and in the other Y ,
say, they are of the same sign. Molyneux[1] demonstrates that the distance between
adjacent points of each of these groups is a constant proportional to the wave-
length. If the functions are free to translate through one another in the direc-
tion of the common wave centre line, but are subject to the constraint that the
locus of each of the intersection points of one of the groups is a straight line
normal to the common wave centre line, then the translation of one waveform
results in the translation of the other in the same or opposite direction, depend-
ing on which group of points is constrained. The ratios of the distances trans-
lated is constant and equal to the ratio of the wavelengths of the two inter-
acting waveforms. In effect a gear ratio is formed. This is the basic principle
of the follower-tooth reduction gear.

In a linear system the common wave centre line is a straight line, whereas
in a system designed for continuous rotation the common wave centre line is a
circle but the functions must each join continuously around this. Although the
mathematical concept is simple the practical problem is to derive engineering
systems whereby the functions can move 'through' each other.

Molyneux's solution to this problem for a linear system (see Fig 2) is to
derive gear profiles which are the envelopes of a circle of radius r whose
centre traces the basic waveforms. Rollers of radius r , sliding in parallel
slots spaced at discreet intervals $\Delta x = 2\lambda/(G + 1)$, or $\Delta y = 2\lambda/(G - 1)$,
where λ and λ/G are the wavelengths of the waveforms, then serve as meshing,
follower-tooth elements by which traction between one gear profile and the other

is obtained. For the equivalent rotary system the common centre line of the basic waveforms is a circle of radius R, centre A, and the wavelengths of the basic waveforms must be such that $(2\pi R/\lambda)$ and $(2\pi RG/\lambda)$ are integers. Gear profiles are derived as for the linear system but the follower-tooth slots are now radial to centre A and are uniformly displaced at angular intervals $\Delta x/R$ or $\Delta y/R$.

For the specific cylindrical-rotary-eccentric configuration analysed in what follows, one of the basic profiles is a circle of radius R eccentric to centre A, and this together with follower-tooth radius and gear-ratio G suffices to define the essential geometry of the reduction gear system.

3 SLIDING CONDITIONS AND BEARING LIMITATIONS OF THE CYLINDRICAL ROTARY FOLLOWER-TOOTH MECHANISM, AND THE ALLEVIATION OF THESE PROBLEMS

The cylindrical rotary design is shown in Fig 3.

The cam is a circular disc mounted on an eccentric journal bearing of the high speed input crankshaft. It need not rotate at input crankshaft speed but at a speed dependent upon the interaction with the roller teeth, cage and outer ring. In theory, assuming that the roller teeth do not slide on the cam or the outer ring, severe sliding must occur between the roller teeth and the slot faces of the radial restraint cage where the high output torque loads are reacted. As the roller teeth are pushed radially outwards by the cam they roll against the cam and outer ring profiles, but their direction of rotation is opposite to that necessary for rolling at the cage slot face. Consequently sliding occurs at the slot face at a speed equal to the sum of the outward radial speed and the peripheral speed of the roller tooth. In practice, due to oil films, machining tolerances and vibration, a small amount of sliding could occur between the roller teeth and the cam and outer ring but the most severe sliding would still occur between the roller teeth and cage slot face, as indicated in Fig 3. The cam rotational speed is slightly greater than the low output speed but the centre of mass of the cam orbits at the high input crankshaft speed. Consequently inertia forces created by the cam mass may be transmitted to the roller teeth and the cam journal bearing as indicated in Figs 4 and 5. Also, the radial torque reaction force is transmitted via the roller teeth and cam to the eccentric journal bearing since the cam is not symmetrically loaded. The resultant load on the cam bearing and main bearings is the vector sum of the torque reaction forces and the cam inertia force. The input crankshaft main bearing loads due to these forces can be reduced to zero if three sets of components are used, ie if the input crankshaft has three eccentric journals upon which are fitted three cams and their bearings

similar to the harmonic drive wave generator arrangement described in Ref 19. The eccentricity of the central journal must be equal to, but disposed diametrically opposite to, the eccentricities of the two adjacent journals and the central set of components must transmit 50% of the power and each adjacent set 25%.

However, for high power transmission at high speed, the loads on the cam bearings would be high, primarily due to the requirement for a small cam eccentricity, and these bearings must have sufficient load capacity to operate for the required number of revolutions. Rolling element bearings have limited fatigue lives and are unlikely to operate successfully in high speed and high power speed reducers of this type. Plain hydrodynamic bearings theoretically have infinite fatigue life and their power losses are similar to those in rolling element bearings at moderately high speeds. Therefore, plain bearings are considered more appropriate for the high power application. The wear likely to occur due to the severe sliding occurring between the roller teeth and the cage slot faces, as indicated in Fig 6, may be alleviated by using an effective lubricant and applying surface film additives. However, substantial frictional heat losses will still occur and the mechanism will suffer from large power loss and low transmission efficiency. It is believed that these problems may be avoided by adoption of the oscillatory bearing concept shown in Fig 7 in which a Rolamite strip is used to control the alignment of the oscillating plain hydrodynamic bearing and the roller tooth acts as the journal of this bearing. The roller tooth protrudes at each end of the plain bearing where it contacts the eccentric cam(s). The plain bearing loss would be much less than that incurred if the roller tooth were allowed to slide against the cage slot face. The Rolamite strip concept is fairly recent and, while it appears to be theoretically sound, experience in applications appears to be limited.

In conclusion, the highest power transmission and minimum power losses should be achieved with cylindrical rotary designs containing three eccentric cams with plain hydrodynamic bearings, and Rolamite strip controlled roller teeth with plain bearings. For low power and low speed applications, simpler versions of this type of mechanism may be acceptable. For example, a cylindrical rotary mechanism containing one eccentric cam with a rolling bearing, and roller teeth without Rolamite strips and plain bearings, could be satisfactory.

4   CONSIDERATION OF PARTICULAR FEATURES

The design techniques used for cylindrical rotary speed reducers and the theoretical performance of a range of sizes of these speed reducers as summarised in the Figures presented at the end of this Report, together with their

application to helicopter main rotor gearboxes, are discussed in the following sections.

4.1 Geometrical features and profile errors

For small cylindrical rotary speed reducers it is shown in Appendix A that if the outer ring waveform is not modified from the simple cosinusoidal form there will be some free movement of the roller teeth. This could lead to the teeth binding, or to chatter, skewing and brinelling of the roller teeth, low fatigue life and high friction losses. Dimensional errors of about 0.2 mm on radius occur for a wave amplitude (2a) of 10 mm and outer ring waveform pitch circle radius (R_{OTR}) of 75 mm. These errors increase as wave amplitude or eccentricity increase and the pitch circle radius decreases. However, this problem is greatly diminished as the pitch circle radius increases provided that

$$\frac{a^2}{2R} < 0.05 \text{ mm}$$

and is completely removed if the outer ring waveform is modified as suggested in Appendix A.

4.2 General strength and mass considerations

It is believed that the triple eccentric system as outlined in section 3 affords the most sensible design for high speed and high power transmission. Main bearing torque reaction loads are negligible and torque loads are shared by three sets of components. For rolling components such as the roller teeth, this affords advantages in fatigue life. However, triple eccentric mechanisms are more complex than single eccentric mechanisms and their power to weight ratio is likely to be lower, especially for low power applications.

The speed reducer components are most heavily loaded at design speed when the eccentric cam inertia force and the torque force are maxima. The inertia force due to the rotation of the centre of mass of the eccentric cam (plus any bearing race housed within it) at eccentricity 'e' from the centre line of rotation of the input crankshaft is determined when the mass and orbiting speed are known, as discussed in Appendix B.

High values should be avoided to prevent excessive forces on the eccentric cam bearings, and an empirical limitation has been assumed, such that the maximum inertia force does not exceed the torque force.

On start-up, torque is required to accelerate the components of the speed reducer and the output load system from zero to design speed and also to

overcome friction. Consequently the eccentric cams and their bearings, roller teeth, cage and outer ring are all loaded. Where plain hydrodynamic bearings are used it is advisable to avoid start-up (or shut-down) wear, by using techniques such as hydraulic jacking, hybrid bearings, surface film additives and solid lubricants.

Where the gearbox weight is to be kept to a minimum, the outer ring should be integral with the gear case with the cage providing the output. Suitable provision must be made to ensure correct lubrication of the roller teeth and their plain bearings, if used.

4.3 Fatigue life

To estimate fatigue life at the roller tooth/inner cam ring/outer tooth ring load bearing faces the tooth is assumed analogous to a roller in a rolling element bearing. Much information on fatigue life, speed and loading of roller bearings is available[2-5] and by comparing with the conditions of a roller tooth in a follower-tooth speed reducer of stipulated power, speed and gear ratio, assuming all rolling contact, it is possible to obtain a value of B10 fatigue life (ie the life which 90% of a large group of identical bearings exceeds under the same operating conditions). Referring to Figs 8 and 9 it can be seen that for a given power, fatigue life of the roller teeth in triple eccentric units increases as speed and outer ring tooth pitch circle diameter increase and gear ratio decreases.

Fig 10 shows the fatigue life of the eccentric cam roller bearings which are fitted on the high speed input shaft. It can be seen that for a given power, life increases as gear ratio and speed reduce and outer ring tooth pitch circle diameter increases. However, adequate life is only achieved if the gear ratio and input speed are no greater than 40:1 and 10000 rev/min respectively. Power capacity at these values is restricted to about 20 kW in a 600 mm unit and 15 kW in a 400 mm unit. At speed ratios less than 10:1 and input speeds less than 1000 rev/min at least 300 kW can be transmitted in a 600 mm unit, although the power capacity of a 400 mm unit is limited to about 120 kW, adequate fatigue life being achieved in each case. For higher power plain hydrodynamic bearings must be used; their limitations are discussed in section 4.5.

4.4 Hertzian stress

Hertzian stress is dependent upon roller tooth diameter which in turn is dependent upon the gear ratio and amplitude of the outer tooth ring waveform. As the gear ratio is reduced, the roller tooth diameter can be increased with the restriction that its radius of curvature must be less than the minimum radius of

curvature of the waveform traced by its centre. As the outer tooth ring diameter
is reduced the radius of curvature reduces unless the wave amplitude is also
reduced. A smaller roller tooth diameter results in higher Hertzian stress and
if the stress exceeds 2100 MPa there is a danger of permanent deformation of the
rolling element assuming normal carbon-chrome bearing steels are used. For a
given power, gear ratio and wave amplitude there is thus a limit to outer ring
tooth pitch circle diameter, below which excessive surface stressing may occur,
leading to very short fatigue life.

4.5 Power loss

Sliding will produce roller tooth wear and power loss. For all sliding
contact design assessments, data from experimental work on sliding and rolling
have been used[6-8]. Sliding occurs (i) to a limited extent between the eccentric
cam, roller teeth and outer ring, and (ii) between the roller teeth and their
radial restriction cage. In case (i) attempts to calculate power loss using
results of experimental work show that for a given power output, the loss (percent)
decreases as speed increases. Mixed film lubrication exists and scuffing,
spalling, abrasive wear and smearing could develop, thus increasing losses and
reducing the fatigue life of rolling components. Fig 11 indicates the case (i)
losses in the central eccentric roller tooth systems of two triple eccentric
units, these central sets of components transmitting 50% of the total power.
It is likely that losses in the outer rotor roller tooth systems would be approxi-
mately equal to the central system losses so that case (i) total losses could
range from about 0.1% to 10%. The conditions used to determine this loss must
be viewed with caution since they are based on limited experimental data.
Sliding in the eccentric cam/roller tooth/outer ring region might be reduced by
optimising clearances; this could involve a pre-loaded system and may result in
lower power losses. In case (ii), the roller teeth rotate at a speed dependent
upon the output speed and at the same time are being forced against the cage slot
faces and are reacting the total output torque. This is a severe operating con-
dition for the roller tooth and cage materials, and mixed film lubrication is
again evident. Surface film additives may alleviate the problem but further modi-
fications to the design are necessary, inevitably leading to complication, weight
increase and possibly more maintenance, since Fig 11 shows that case (ii) loss is
high (7% to 17%) at all speeds. By using an oscillatory plain hydrodynamic
journal bearing (Fig 7) case (ii) losses may be reduced appreciably in the low
and medium speed range. However, it should be noted that this conclusion is
reached assuming the bearings are at their most heavily loaded condition and
employing normal plain bearing theory.

No theory is available to determine these losses for bearings that are oscillating as well as rotating, ie where the direction and value of load are varying cyclically; if the lubricant laminar flow is disturbed due to oscillatory motion, turbulence may occur and this can produce a marked increase in power loss. However, it is considered that the Rolamite strip control design provides the best practical solution to the controlled radial location of roller teeth. Fig 12 indicates the losses in the central eccentric oscillatory bearings of two sizes of triple eccentric units, where each central eccentric cam transmits 50% of the total power. Plain bearing power loss theory[9] was used to calculate the losses. The additional losses in the two sets of outer eccentric oscillatory bearings are about 1.5 times greater than the latter. Thus the total oscillatory journal bearing losses range from less than 1% at low and medium input speeds to about 45% at very high speed. The latter value is academic since it may be shown that the design is becoming impractical. A realistic maximum is about 10%.

Rolling bearing power loss theory[10] shows that the loss in the central cam rolling bearing of a 373 kW unit varies as shown in Fig 13 from about 10% at low input speed (1000 rev/min) and high gear ratio (G = 100) to 0.25% at very high input speed (30000 rev/min) and low gear ratio (G = 10). At 5000 rev/min input speed, power loss is about 5% at high gear ratio and falls to about 1% at low gear ratio. However, it has been indicated that rolling bearings have inadequate fatigue life if power in excess of about 300 kW is transmitted at input speeds and gear ratios in excess of 1000 rev/min and 10:1 respectively. Consequently they must be rejected in favour of plain hydrodynamic journal bearings. The latter tend to have slightly higher power losses as can be seen by comparing their values in Fig 14 with Fig 13. The convolutions of the individual power curves are associated with variations of load limitations of plain bearings with speed, power level, bearing size and diametral clearance.

In Fig 14, where journal bearing parameters have been optimised for unit powers in excess of 186 kW (for a typical gearbox oil), percentage loss ranges from less than 1% to 22% for the central eccentric cam, the high values being associated with low speed and power levels. The total power loss of the three journal bearings is about 2.5 times these values (ie from about 1.25% to 28% power loss), if the outer eccentric plain bearings are optimised. In all cases the turbulent region is avoided. Plain bearings do not have the fatigue life problem associated with rolling element bearings although they may wear if adequate lubrication is not provided on start-up or shut-down. Soft wear-reducing materials used as a bearing wall substrate may provide lower friction

coefficients thus presenting a simple solution. An alternative solution is hydraulic jacking, ie the feed of high pressure oil to pockets in the plain bearings so that the bearing and journal surfaces are separated by oil films before start-up. Auxiliary power supplies to produce high pressure oil may be required, which can be regarded as disadvantageous in terms of increased complexity, weight and maintenance. A further solution involves the use of hybrid rolling element/plain bearings although this may have the same disadvantages.

It is generally true that for the same duty, the weight of a plain hydrodynamic journal bearing is less than that of a rolling element bearing, neglecting pumping equipment. Oil pumps are always required for current main rotor gearbox oil circulation and it is likely that the requirements of plain hydrodynamic journal bearings would be met by such pumping systems thus discounting the need for further oil feed equipment. The vulnerability aspect is more critical to the plain bearing than to the rolling element bearing due to the comparatively better survival prospects of the latter under oil starvation conditions.

4.6 Main bearings

In single and double eccentric systems main bearing loads may be minimised by the addition of suitably positioned balance weights; in a triple eccentric system the weight of the central eccentric should be equal to the sum of the weights of the two outer eccentrics (of equal weight), and it should be ensured that the outer eccentrics have the same eccentricity and are diametrically opposed to the central eccentric and spaced equally from it in the axial direction. When these conditions apply, main bearing loads and power losses are small permitting the use of rolling element bearings with adequate fatigue life.

4.7 Disc friction

The disc friction occurring in the speed reducer is due to the rotation of the eccentric cams in the gearbox oil. For a gearbox completely filled with oil friction loss is proportional to the cube of the eccentric cam peripheral velocity. In practice only about 5% of the gearbox air volume is filled with oil and calculations suggest that disc friction losses will be much less than 1% of transmitted power.

4.8 Rolamite strip controlled teeth

In an effort to reduce the sliding phenomena mentioned in section 4.5 it is suggested that the roller tooth could act as the journal of a bearing. However,

the bearing must be located in the radial restraint cage so that it can oscillate radially without sliding against the cage wall. The best solution is considered to be that where a Rolamite strip is wrapped around the bearing and fixed at its ends to the cages slot faces (Fig 7). Such a design requires oil feed to all the oscillatory bearings. Furthermore, since the radial restriction cage is normally the output member of the speed reducer it also rotates and the oscillating bearings orbit the centre line of the gearbox. Consequently the oil feed system is complex and efficient radial or axial seals are needed.

4.9   Other operational problems and possible solutions

Roller tooth skewing due to the interaction of the eccentric cam/roller tooth/cage and outer ring at high speeds and loads in mixed film lubrication conditions may occur. This could be aggravated by thermal distortion, general vibration and minor manufacturing errors. However, the use of lightly pre-loaded hollow roller teeth designed so that they are allowed to distort minutely (ie a cross section is permitted to distort as an oval) may alleviate these problems. A small amount of input shaft centre line/cage centre line misalignment is allowable without seriously affecting performance since journal bearings have typical clearances of about 0.05 mm. Also the cage may be designed so that circumferential distortion is negligible at high torque loading so this should not promote roller tooth skewing. For satisfactory speed reducer operation it follows that careful attention should be paid to the detailed specification of manufacturing tolerances, surface finishes and materials. This is especially so for the eccentric cam and outer ring surface profiles, the Rolamite strips and roller teeth. For example, the eccentric cams and outer rings may be considered as the inner and outer races of rolling element bearings and as such should be produced from bearing steels and have similar surface finishes. The roller teeth may also be considered as the rollers in rolling element bearings and should be manufactured from ball or roller bearing steels. A satisfactory Rolamite strip material is Sandvik 11R51 stainless steel which has ten times the fatigue life of normal stainless steels[11].

Because the speed reducer contains rolling elements and plain hydro-dynamic journal bearings it is necessary for the components to be machined to a high degree of accuracy in order that load sharing is correctly disposed throughout the roller tooth systems. Such a mechanism is likely to be more sensitive to foreign body entrainment in the lubricant/cooling oil than conventional gearboxes containing involute tooth gears. Efficient fine mesh filtration systems are necessary for the oil feed system.

Finally, for the helicopter application, due account should be taken of the effects of rotor blade bending moments reacted in the gearbox.

4.10 Helicopter application

Most helicopter lifting rotors are driven by free power gas turbine engines (one or two per main rotor) via reduction gearboxes. The rotor blade tip speeds are restricted to about 210 m/s so the rotational speed is restricted to a few hundred revolutions per minute, typically 250 rev/min. The engine free power disc speeds range from 10000 rev/min for large engines to 30000 rev/min or higher for small engines. The drive gear trains must therefore provide reduction ratios up to about 90:1. Normally the first reduction (about 5:1) occurs in a single or double epicyclic gear train or a helical spur gear train integral with the gas turbine, and all further reductions, usually about 18:1, are provided by intermediate and/or main rotor gearboxes. An exception is the Sikorsky SH3D Sea King whose engine output shaft speed is about 19000 rev/min. In this case the first reduction gearbox is not fitted to the engine but is integral with the main gearbox and the overall speed ratio is 93:1.

A study of current basic gearbox weight[12] shows a range of power to weight ratios of 2.9 kW/kg to 12.5 kW/kg (although these two values are extreme cases) and an average value of about 4.1 kW/kg. In this study a basic gearbox has been deemed to comprise all the components (gears, bearings and gear cases) necessary to transmit the drive from the engine to the main rotor but shafts and auxiliary gear drives have been excluded.

A design study of the follower tooth reduction gear has been made (see Appendix C) based on the requirements for a typical current helicopter main rotor gearbox. The mechanism has been designed to transmit 1492 kW (2000 hp) at 5000 rev/min input speed with a gear ratio of 20:1 using an outer tooth ring diameter of 500 mm. Rolling element main bearings are used and the eccentric cam and roller tooth oscillatory bearings are plain hydrodynamic. The total theoretical power loss is calculated to be 4.5% and the combined roller tooth B10 fatigue life is estimated to be 3700 hours. The power to weight ratio is calculated to be 5.5 kW/kg. Sectional views of the speed reducer are shown in Figs 15 and 16.

Helicopter lifting rotor shaft and gearbox centre lines are essentially coincident and vertical, and the turbine engine centre lines are horizontal in most cases. Consequently, right angled drives incorporating spiral bevel gears are necessary and normally form one of the speed reducing stages. From general experience the power to weight ratio of a reasonably high speed spiral bevel gear

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pair plus their mounting bearings and surrounding gear case is approximately
20 kW/kg. The follower tooth reduction gear considered here has a speed reduc-
tion ratio of 20:1 and its centre line must be coincident with the vertical
lifting rotor shaft centre line. If combined with a first stage 1:1 bevel gearbox
the estimated value of total basic transmission power to weight ratio is 4.3 kW/kg
and power loss is likely to be more than 5%.

NOTE: Lifting rotor shaft bending moment effects have not been included and
in practice another large output bearing may be required to react the bending
moments (see dotted extension in Fig 15); this could reduce the power to weight
ratio to about 4.1 kW/kg.

4.11 Intermediate transmissions

Main gearboxes with high speed ratios require high input speeds. Since
spiral bevel gears are required with horizontally disposed turbine engines, pitch
line velocities are high and high precision quality gears are needed to minimise
dynamic loading. Gear support bearings may be large and in some cases rotate
near to their suggested maximum speeds. With vertically disposed turbine engine
output shafts high performance flat belts offer a light and cheap intermediate
drive. Bearing weight is low and fatigue life above nominal requirements. It is
indicated in Ref 13 that either of these systems could be used with a follower
tooth reducer having a speed ratio of 20:1. However, as the ratio is increased
for a fixed output rotor speed, the power capacity of such systems reduces.

4.12 Component materials

Where possible the material used for the components in the design study are
the same as used in conventional transmissions. For instance:

Magnesium alloy RZ5 (density 1.81 g/cm^3) is used for the gear case but steel
(7.9 g/cm^3) is used for the outer rings which are built into the gear case. Steel
is used for the main bearings, input shaft, roller teeth and torque reacting faces
of the cage. Titanium-manganese (4.5 g/cm^3) is used for the remainder of the cage
and the output shaft. Titanium and steel are used for the eccentric cams. The
Rolamite strip material is Sandvik 11R51 stainless steel.

The assumption that these materials are used enables a fair weight compari-
son to be made with conventional transmissions.

4.13 Ancillary systems

In current helicopter transmissions, tail rotor and auxiliary drives (for
pumps, alternators etc) are usually taken from the main gearbox. This is likely

to produce greater complication and weight increase for a follower tooth reduction gear than for a conventional gearbox. The configuration of gears within a conventional gearbox allows additional auxiliary drive pinions to be arranged easily to mesh with the gears of the basic transmission. The arrangement of the follower tooth reduction gearbox requires gears to be fitted to the basic transmission shafts which then drive the pinions of auxiliary systems.

4.14 Some aspects of manufacture and maintenance

Although the follower tooth reduction gear is basically of simple geometry, high accuracy machining and high quality surface finishes are essential and the basic design is complicated by the necessary addition of oscillatory bearings. This implies that manufacture, quality control and assembly costs may not be competitive with those for conventional systems. Spur, helical and spiral bevel gears would still be required for auxiliary drives and it follows that there would be an increase in the variety of jigs, fixtures and machine tool operations necessary to manufacture the total system.

The components of the speed reducer that will suffer from fatigue (similar to surface fatigue in rolling element bearings) are the eccentric cams, the roller teeth and the outer ring and the fatigue life of the speed reducer is essentially equivalent to the fatigue life of these components. Conventional gearboxes use rolling element bearings to mount the gear shafts in the gear cases. Normally the gears have much longer operational lives than the bearings and, after a strip-down inspection, usually only the bearings and seals are replaced when the gearboxes are rebuilt. The cost of refurbishing a follower tooth reduction gearbox may thus be greater than that for a conventional gearbox because it may be necessary to replace the eccentric cams, roller teeth and outer rings as well as the main bearings.

On the whole it is considered that conventional transmissions are easier to manufacture, are more robust than the follower tooth reduction gear and should have much longer operational lives.

5 CONCLUSIONS

5.1 The cylindrical rotary mechanism is considered to be more appropriate to continuously operating high power and high speed drives than any of the other follower tooth mechanisms devised by Molyneux[1] because the rolling elements of the latter, such as the roller teeth, generally operate at higher sliding and rotational speeds and thus more severe sliding wear is liable to occur. However even the cylindrical rotary mechanism suffers from severe sliding at the contact

region between the roller teeth and their radial restriction cage slot faces, but the geometry of the mechanism is such that this can be alleviated appreciably.

5.2   The incorporation of an oscillatory plain hydrodynamic bearing housed in a Rolamite strip with the roller tooth acting as the journal may successfully overcome the severe sliding mentioned in 5.1 although at the expense of increased complexity and weight. This modification (or a similar alternative) must be included in high speed and high power drives.

5.3   Multiple eccentrics should be used in order to moderate the bearing and roller tooth loading. It is believed that the best compromise considering complexity, efficiency, weight and life is a drive containing three eccentric cams, where the central cam transmits 50% of the total power and the two outer cams each transmit 25% of the total power.

5.4   In the cylindrical rotary design the eccentric cam circular profile is only valid geometrically for outer ring cosinusoidal waveforms if

$$\frac{a^2}{2R} < 0.05 \text{ mm}$$

(ie large diameter outer ring waveform pitch circle diameters with small amplitude waveforms and thus small eccentricity). If $a^2/2R > 0.05$ mm the outer ring waveform requires modification to avoid geometrical errors.

5.5   It is necessary to make a compromise between eccentricity, outer ring waveform pitch circle diameter, and loading of the rolling elements in the rotary design since for a given transmission power and gear ratio the load increases with decreasing eccentricity. Inertia effects and roller tooth diameter restrict the value of eccentricity. If rolling element bearings are used as the cam bearings in triple eccentric units, adequate fatigue life can only be achieved for moderate power transmission, if input speeds and gear ratios are less than 10000 rev/min and 40:1 respectively.

5.6   For high power and speed applications it is necessary to use plain hydrodynamic bearings for the eccentric cams of the rotary design to avoid the fatigue problems associated with rolling element bearings. However, wear can occur in plain hydrodynamic bearings during the start-up and shut-down phases of operation and methods of alleviating this (eg hydraulic jacking, surface coatings or the use of hybrid bearings) should be considered.

5.7   It is difficult to predict the performance of the roller tooth plain hydro-dynamic oscillatory bearing and the extent to which sliding occurs between the roller teeth, eccentric cam and outer ring at high input shaft speed, particularly in the presence of oil films. Performance can be influenced by factors not apparent from a theoretical analysis. For example, skewing of roller teeth; thermal distortion; vibration; manufacturing errors; the effect of journal mis-alignment in roller tooth and eccentric cam bearings; and the associated oscilla-tory effects. Flexing hollow roller teeth may reduce these effects and provide better load sharing.

5.8   For satisfactory speed reducer operation the detailed specification of manu-facturing tolerances, surface finishes and the materials of the eccentric cams, outer rings and roller teeth should be similar to those for rolling element bearings to ensure adequate surface fatigue life.

5.9   The speed reducer contains rolling elements and plain hydrodynamic bearings and the roller teeth are not sealed from the environment within the gearbox. Thus these contacting zones are open to ingress of particles which could cause serious surface deterioration. It is therefore important to provide fine mesh oil filtration systems and because of the nature of the mechanism these will probably be fairly complex.

5.10   Generally the follower-tooth reduction gear is not competitive with current helicopter main rotor basic transmissions. Theoretically a design can be produced which has an overall fatigue life (and time between overhauls) in excess of 2000 hours and a transmission efficiency of 95.5% compared with a transmission efficiency in the range 95-98% for existing gear transmissions. However, for reasons mentioned in 5.7, efficiency in practice may be less than 95.5%. The power to weight ratio of the basic follower-tooth speed reducer is about 5.5 kW/kg if an angled drive is not included. However, the addition of an intermediate angled drive stage reduces its power to weight ratio to about 4.3 kW/kg and its efficiency by about 0.5%. Lifting rotor shaft bending moment effects have been excluded and in practice another large output bearing may be required to react the bending moments. This could reduce the power to weight ratio to about 4.1 kW/kg, which is comparable to the average value for existing gearboxes, while the efficiency is appreciably lower.

5.11   The addition of tail rotor and ancillary drive take-offs will increase complexity and weight compared with a conventional helicopter transmission system.

5.12 Although components are geometrically simple, high manufacturing standards, quality control, assembly difficulties, and the need to provision a variety of machine tooling is liable to reflect adversely on costs.

Refurbishment of the follower-tooth reduction gear probably entails replacement of the eccentric cams, outer rings, roller teeth, main bearings and seals, whereas conventional systems normally require only main bearing replacement and very rarely the replacement of a gear wheel.

Appendix A

GEOMETRICAL FEATURES OF THE FOLLOWER-TOOTH REDUCTION GEAR

A.1 Profile equations and errors in the simple cylindrical rotary design

The simplest functions which can be used to produce the required waveforms are cosinusoidal and the most elementary form of inner profile for a continuous rotation system is a *single* cosine wave superposed on a base circle. In polar coordinates, referred to an origin at the centre of the base circle of radius R , say, the equation of such a profile is

$$r = R + a \cos \theta \tag{A-1}$$

where 2a is the amplitude of the superposed cosine wave. This design is not considered explicitly by Molyneux[1] and all his concentric designs are concerned with inner profiles having *multiple* cosine waves superposed on the base circle, ie functions of the form

$$r = R + a \cos n\theta \tag{A-2}$$

where n is an integer greater than or equal to 2.

However, Molyneux approximates to the concentric design having an inner profile with a single cosine wave superposed on a base circle by introducing the eccentric rotary design. In this case his inner profile is a circle with its centre offset from the centre of the base circle of the outer profile, which is the centre of rotation of the unit, by an amount equal to half the amplitude of the superposed cosine wave. The great advantage gained from this offset arrangement is that it allows the insertion of a bearing *concentric* with the inner circle profile thereby ensuring that the circular profile rotates at a much lower speed than the input shaft, which ensures low rotational and sliding speeds of the follower-teeth.

The eccentric design is only an approximation to the most elementary form of concentric design inner profile because the locus of a point on the circumference of a circle rotating about an offset centre is, referring to Fig 17, given by

$$R^2 = r^2 + a^2 - 2ar \cos \theta \quad ,$$

ie

$$r = a \cos \theta + (R^2 - a^2 \sin^2\theta)^{\frac{1}{2}} \tag{A-3}$$

or

$$r = R + a \cos \theta - \frac{a^2}{2R} \sin^2\theta + 0\left(\frac{a^4}{R^3}\right) . \tag{A-4}$$

Thus, if the outer profile is to have the form

$$r = R + a \cos G\theta , \tag{A-5}$$

where G = the speed ratio, being an integer $\geqslant 2$, the offset circle approximation is only valid if the term $(a^2/2R) \sin^2\theta$ is negligible in equation (A-4) (ie equation (A-4) approximates to equation (A-1)). The permissible magnitude of this term is a matter for engineering judgement and is associated with practical manufacturing tolerances. If, in the course of initial design work, $a^2/2R$ is chosen to be greater than some arbitrary figure, say 0.05 mm, as a tolerance limit, then the outer profile must be corrected to reduce the error from the off-set circle approximation. The corrected form follows from the fact that equation (A-3) may be written

$$r = \left(R - \frac{a^2}{4R}\right) + a \cos \theta + \frac{a^2}{4R} \cos 2\theta + 0\left(\frac{a^4}{R^3}\right) \tag{A-6}$$

which has the corresponding outer profile form

$$r = \left(R - \frac{a^2}{4R}\right) + a \cos G\theta + \frac{a^2}{4R} \cos G\theta + 0\left(\frac{a^4}{R^3}\right) . \tag{A-7}$$

In general, the terms of $0(a^4/R^3)$ are negligible.

In many cases, particularly for large units, it is unnecessary to use the modified form of outer profile, but for small units modification may be essential.

In practice, roller teeth separate the eccentric cam and outer ring profiles such that equations (A-6) and (A-7) represent the traces of the centre of the roller teeth with respect to the eccentric cam and outer ring respectively. The envelopes traced by the roller teeth are then the eccentric cam and outer ring profiles.

The equation of the outer ring envelope is

$$r^2 = \left[(R + a \cos G\theta)^2 + R_{RT}^2 \right] + 2R_{RT}(R + a \cos G\theta)$$

$$\times \left[\sin\theta - \cos\theta \left\{ \frac{aG \tan\theta \sin G\theta - (R + a \cos G\theta)}{aG \sin G\theta + (R + a \cos G\theta) \tan\theta} \right\} \right]$$

$$\times \left[1 + \left\{ \frac{aG \tan\theta \sin G\theta - (R + a \cos G\theta)}{aG \sin G\theta + (R + a \cos G\theta) \tan\theta} \right\}^2 \right]^{-\frac{1}{2}}$$

and that of the eccentric circular cam is

$$r^2 = (R - R_{RT})^2 - a^2 + 2ar \cos\theta \quad .$$

However, numerically controlled machining of these profiles only requires the stipulation of the coordinates of the centre of the roller teeth (from equations (A-6) and (A-7)) if the cutters or grinding wheels have the same diameter as the roller teeth.

A.2 Geometrical limitations of profiles and roller teeth

The roller follower-tooth radius R_{RT} must always be smaller than the minimum radius of curvature (ρ_0) of the higher frequency waveform traced by the roller follower-tooth centre. This ensures geometrically correct operation and continuous contact (see Fig 18). The locus of the centre of the roller tooth is a cosinusoid whose centre line is a circle of radius R (see equation (A-5)), and the minimum radii of curvature occur at the maximum amplitude positions, where $r = R + a$ and $r = R - a$. The radius of curvature is given by

$$\rho = \frac{\left[r^2 + \left(\frac{dr}{d\theta} \right)^2 \right]^{\frac{3}{2}}}{\left[r^2 + 2\left(\frac{dr}{d\theta} \right)^2 - r \frac{d^2 r}{d\theta^2} \right]} \quad . \tag{A-8}$$

From (A-5) and (A-8)

$$\rho = \frac{\left[R^2 + a^2 + 2aR \cos G\theta + a^2 \sin^2 G\theta (G^2 - 1)\right]^{\frac{3}{2}}}{\left[R^2 + a^2(G^2 + 1) + aR \cos G\theta (G^2 + 2) + a^2 \sin^2 G\theta (G^2 - 1)\right]}$$

which has a minimum value, at radius $r = R + a$, of

$$\rho_0 = \frac{(R + a)^2}{(R + a) + G^2 a} \tag{A-9}$$

and a value, at $r = R - a$, of

$$\rho_s = \frac{(R - a)^2}{(R - a) - G^2 a} \quad . \tag{A-10}$$

Referring to Fig 18 if the minimum radius of curvature ρ_2 of the outer ring wave profile is to be finite, than R_{RT} must be less than ρ_s as well as being smaller than ρ_1. Also, in order to avoid any possibility of 'trapping' or 'binding' of the roller tooth between the cage radial restraint and the outer ring waveform, R_{RT} should be larger than the wave amplitude. It is logical therefore to design for the largest possible value of R_{RT} consistent with the above limitations.

Let

$$\rho_s \geq R_{RT} \geq 2.5a \quad . \tag{A-11}$$

From (A-10) and (A-11), for given gear ratio G and (a/R), ρ_0 or ρ_s can be determined, or conversely (a/R) can be found for various values of G and the limiting value $\rho_s = 2.5a$; thus, dimensional limitations of the system geometry can be developed. Note that in Figs 19 and 20, if ρ_0 and $\rho_s < 10a$ then for the same R and G, $\rho_0 \cong \rho_s$. Table 1(a) is collated for the condition $\rho_0 \cong \rho_s = R_{RT} = 2.5a$. However at low gear ratios ($G = 10$ and 20) the outer ring waveform has a large amplitude (half amplitude a) and the eccentricity ($e = a$), of such units, especially when D_{OTR} (the outer ring waveform pitch circle diameter) is large, would appear excessive. In such cases it is advisable to adopt the condition $\rho_0 \cong \rho_s > 2.5a$ in order to reduce e and a, also allowing $R_{RT} > 2.5a$.

However, the condition $\rho_s = R_{RT}$ should not be allowed since it infers that the outer ring waveform inner peaks have radii of curvature ρ_2 approaching zero; such a condition would tend to produce fatigue, intermittency of roller tooth rolling action, break-up of lubricant film etc. It is thus advisable to ensure $\rho_s > R_{RT}$ but also allow R_{RT} to be as large as possible providing $R_{RT} < \rho_1$.

A further restriction on the dimension R_{RT} arises because it depends upon the number of roller teeth, their pitch radius R and the cage rib width necessary to react the output torque (see Fig 21). Since the choice of cage rib width is fairly flexible and its minimum value is dependent upon output torque, for this assessment it will be taken equal to D_{RT}. Therefore

$$2\pi(R_{OTR} - R_{RT}) = 2R_{RT}n_{RT} + 2R_{RT}(n_{RT}-1)$$

where $2R_{RT}n_{RT} \triangleq$ arc length of total number of rollers

$2R_{RT}(n_{RT}-1) \triangleq$ arc length of the total number of cage rib widths.

Therefore

$$R_{RT_C} \triangleq \frac{\pi R_{OTR}}{\left[(2n_{RT} - 1) + \pi\right]} \qquad \text{(A-12)}$$

From a strength limitation point of view it is important to note that the output torque is never reacted by more than 50% of the total number of roller teeth, and thus up to 50% of the cage ribs also (the actual percentage depending on inertia effects).

> Example: Choose $D_{OTR} = 600$ mm   and $G = 10$.

For $\rho_s = R_{RT} = 2.5a$, Table 1(a) shows that $a = e = 16.2$ mm. This eccentricity is excessive and for practical reasons it is also necessary to choose $\rho_s > R_{RT} > 2.5a$.

Choose the value R_{RT_C} given by (A-12), ie

$$R_{RT_C} \triangleq \frac{300\pi}{(2 \times 9 - 1) + \pi} = 47 \text{ mm} .$$

Also

$$R_{OTR} = R + R_{RT} \qquad \text{therefore} \qquad R = 300 - 47 = 253 \text{ mm} .$$

Choose $a = e = 5.0$ mm . Then

$$R = \frac{253}{5} \times a = 50.6a \quad .$$

From Figs 19 and 20, $\rho_s = 47a = 235$ mm, and $\rho_0 = 17a = 85$ mm. The inequality is thus satisfied.

It should be remembered that Table 1(a) shows the limiting values when $\rho_s = R_{RT} = 2.5a$, and in practice the condition $\rho_s > R_{RT} > 2.5a$ should be observed. Table 1(b) has been obtained by determining R_{RT} from (A-12) (ie R_{RT_C}), the upper limit value for this assessment (R_{RT} could be larger but cage rib width would be reduced).

Comparison of Tables 1(a) and (b) shows that at high gear ratio, little change in R_{RT} occurs, but as G falls to ten, there is up to about 17% increase in R_{RT} . Since in most cases ρ_2 is still approximately zero it is necessary to reduce a . This has an adverse effect on roller tooth life since for a given torque and roller tooth PCD, angle α is reduced and P_T and RF increased. Table 1(c) has been formulated for $\rho_s = 4a$ and the modified values of R_{RT} (ie R_{RT_C}). In these cases $\rho_s > R_{RT} > 2.5a$, the modulus of ρ_2 being greater than zero. In the limit $\rho_1 = -\rho_2$ as R_{RT} and a approach zero, but in the interests of high roller tooth fatigue life, they should be large enough to keep Hertzian stress as low as possible. Table 1(c) indicates that at high gear ratios (ie G = 40 to 100) little is gained in modifying the quantities further since a is small. Further reduction of a would increase the magnitude of ρ_2 , and the torque force P_T would be increased, causing high RF values. At gear ratios below 40, $|\rho_2|$ could be increased at the expense of increased P_T and RF since a is large and could be reduced sensibly. Eccentricity would be reduced which would effectively reduce the inertia of oscillating components (such as the eccentric cam and roller teeth).

Finally it should be noted that fatigue life and power loss calculations are based on the dimensions in Table 1(c).

Appendix B

STRENGTH AND WEIGHT OF SPEED REDUCER COMPONENTS

B.1 Eccentric cam

For a given transmission power, input speed N_I and eccentricity e
($= a$, waveform half amplitude), the torque force P_T (see Fig 4) acting at the
centre of mass of the cam and the inertia force IF of the latter acting at its
centre of mass (but at right angles to P_T) will produce stresses in the cam (see
shear areas in Fig 22) since the power is transmitted through it. The cam must
therefore have sufficient strength to transmit the power. Now the results of the
design studies for different sizes, powers and gear ratios have been shown in
terms of fatigue life and power loss in Figs 8 to 14 and as the special case of
the helicopter main gearbox design study in Appendix C. These results arise by
way of the arbitrary condition that the inertia force IF must never exceed the
maximum torque force P_T. Thus a limitation is set on the weight of the cam
(which includes any part of a bearing fixed in the cam bore) for a given e and
N_I. As input speed N_I approaches 30000 rev/min and eccentricity increases, the
allowable weight becomes very small so as to be unrealistic. Thus there is also
a speed limitation due simply to weight considerations for given eccentricity.

The upper limit of weight is set at 45 kg (for single or multiple cam sets
plus the bearing(s) in the cam bore(s)) and the lower limit depends on strength
and feasible design considerations. Thus, the limitations are

$$\left. \begin{array}{c} \text{IF} \leqslant P_T \\ \\ \text{maximum total cam weight} \leqslant 45 \text{ kg} \end{array} \right\} \tag{B-1}$$

Calculations show that for unit sizes $D_{OTR} = 400$ mm to 600 mm,

(1)   the inertia force only begins to approach the torque force where
$N_I \geqslant 5000$ rev/min at 186 kW and $G \leqslant 20$;

(2)   the inertia force only begins to approach the torque force where
$N_I \geqslant 10000$ rev/min at 1492 kW and $G \leqslant 20$.

B.2 Roller tooth cage strength

Cage distortion must be limited to avoid adverse effects on roller tooth
operation, ie misalignment of roller teeth leading to binding, wear and lower
efficiency. Also the web section shear area must be large enough to withstand
the shear forces due to output torque.

For a helicopter application, the output speed (lifting rotor speed) would be in the range 200-400 rev/min. The output torque of the follower-tooth speed reducer will be transmitted through the cage and is given by

$$T_0 = \frac{60 \times \text{power (watts)}}{2\pi N_0} \quad \text{Nm} \quad . \tag{B-2}$$

The circumferential deflection and shear stress of the cage can be determined from the torsion equations

$$\frac{T_0}{J} = \frac{\tau_0}{y_0} = \frac{G_0 \theta_0}{\ell_0} \quad . \tag{B-3}$$

B.3   Web shear strength and area

From particular cases examined and Fig 23 it can be deduced that the minimum cage rib shear area is not less than 18% of the cage wall cross sectional area, ie with the worst loading condition $P_T = IF$, driving arc FKEH is 135° ($3\pi/4$ rad), and since roller teeth take up a maximum of 50% of this arc, the minimum total cage rib width (Fig 23) is equivalent to 50% of arc length FKEH (ie $3\pi/8$ rad).

For small r_0/D_0, the cage cross-sectional shear area $= \pi D_0 r_0$. Thus,

$$\text{the minimum cage rib shear area} = \left(\frac{3\pi}{8 \times 2\pi}\right) \times \pi D_0 r_0$$

$$= 0.18 \, \pi D_0 r_0 \quad . \tag{B-4}$$

B.4   Cage mass

Cage weight can be considered to be the sum of:

(1)   a cylinder with outer diameter D_0, wall thickness r_0 and length ℓ_0 (the length depending on roller tooth length, the number of eccentric cams and their spacing), and

(2)   two end plates with diameter D_0 and thickness t_0.

Because slots are required in the cage wall to house the roller teeth, and the wall thickness at these zones has to be increased to allow a full bearing surface against which they can roll, it is considered that there is no net weight increase. The roller tooth-Rolamite strip mass is not considered as part of the cage mass.

Appendix C

HELICOPTER MAIN GEARBOX DESIGN STUDY

In the discussion section of Ref 1 it is stated that the follower tooth reduction gear may have "applications for reduction gears in transmission systems, for example, helicopter main and tail rotor gears, tank and armoured car transmission systems and marine turbine-propeller gears" etc.

The current interest is in the development (for helicopter applications) of an efficient high speed, high power gear reduction unit which is to have a time between overhauls (TBO) of at least 2000 h.

A follower tooth reduction gear is now considered for this purpose, the design analysis for a helicopter main gearbox application being shown in the following sections.

C.1 Design requirements

Transmission power = 1492 kW (2000 hp)

Input speed, N_I = 5000 rev/min

Output speed, N_0 = 250 rev/min .

C.2 Initial considerations

The gear ratio G = 20 and a B10 fatigue life of 2000 h is required. A 'triple eccentric' design is proposed since it can be arranged that main bearing loads are negligible throughout the operating range.

The minimum size of unit that appears capable of transmitting the power for the required gearbox life is one with D_{OTR} = 500 mm, the controlling factor being roller tooth B10 fatigue life.

C.3 Roller tooth B10 fatigue life and Hertzian stress

The design of the roller tooth is such that it has constant diameter throughout its length and its central portion, of length L_0 , acts as the journal of the oscillatory plain hydrodynamic bearing; the outer portions, each of length ($L_{RT}/2$) roll between the eccentric cam and the outer ring. The roller tooth total length = $L_0 + L_{RT}$.

Referring to Fig 8 for a triple eccentric 1492 kW unit with D_{OTR} = 600 mm, N_I = 5000 rev/min and G = 20, the B10 fatigue life L_h = 5500 h. For the same power, speed and gear ratio requirements, but with D_{OTR} = 400 mm, the B10 life is 390 h.

A triple eccentric system with $D_{OTR} = 500$ mm and $G = 20$ yields the following:
From (A-12)

$$R_{RT_C} = \frac{\pi \times 250}{(38 - 1 + \pi)} = 19.5 \text{ mm} \quad .$$

From Fig 20 and with $\rho_s = 4a$, $R = 40a = R_{OTR} - R_{RT_C}$, therefore

$$a = \frac{250 - 19.5}{40} = 5.75 \text{ m} \quad (= \text{eccentricity e}) \quad .$$

Referring to Fig 6 it can be shown that

$$\alpha = \frac{\pi}{2} - \gamma \quad \text{and} \quad \tan \gamma = \frac{R}{aG}$$

therefore $\tan \alpha = \dfrac{aG}{R} = \dfrac{5.75 \times 20}{250} = 0.46$

therefore $\alpha = 24.7^{\circ}$.

The central cam torque force

$$P_T = \frac{60 \times \text{power (watts)}}{2\pi e N_I}$$

$$= \frac{60 \times 1492000 \times 0.5}{2\pi \times 5.75 \times 10^{-3} \times 5000} = 25 \times 10^4 \text{ N} \quad .$$

With eccentric cam mass = 22.5 kg, the central cam inertia force

$$IF = me\omega^2$$

$$= 22.5 \times 5.75 \times 10^{-3} \times \left(\frac{5000 \times 2\pi}{60}\right)^2 = 3.55 \times 10^4 \text{ N} \quad .$$

Referring to Fig 4 or 5, the resultant load on the central cam bearing is

$$W = 10^4(25.0^2 + 3.55^2)^{\frac{1}{2}} = 25.25 \times 10^4 \text{ N}$$

acting at 82° to the line of eccentricity.

From Fig 6, Stribeck[14] and information in Appendix A, it can be shown that the maximum reaction force

$$RF = \left(\frac{5W}{n_{RT}}\right) \csc\left[\tan^{-1}\left(\frac{- R_{OTR}}{aG}\right)\right]$$

$$= \frac{5 \times 25.25 \times 10^4}{19} \csc\left[\tan^{-1}\left(\frac{- 250}{5.75 \times 20}\right)\right]$$

$$= 7.5 \times 10^4 \ N \ ,$$

(related to the central cam).

Let

$$L_{RT} = 2D_{RT} = 78 \ mm \ .$$

(a) The central cam roller tooth maximum Hertzian stress, S , can be obtained from the equation

$$S = 0.591 \left[\frac{(RF)_{max} \times E}{L_{RT}D_{RT}}\right]^{\frac{1}{2}}$$

$$= 0.591 \left[\frac{7.5 \times 10^4 \times 21 \times 10^{10}}{78 \times 39 \times 10^{-6}}\right]^{\frac{1}{2}} = 1340 \ MPa \ .$$

(b) Similarly the outer cam roller tooth maximum Hertzian stress is

$$S = 950 \ MPa \ .$$

The basic dynamic capacity (C) for radial roller bearings[4,5] is given by

$$C = f_0(L_{RT}')^{0.78}(n_{RT})^{0.75}(D_{RT})^{1.07}(i \cos \alpha_2)^{0.78} \ kg \ ,$$

where $(i \cos \alpha_2)^{0.78} = 1$ for parallel rollers ($\alpha_2 = 0$, $i = 1$).

The bearing factor f_0 can be derived from the diameter ratio γ_c as

$$f_0 = \left[12.9\gamma_c^{0.222}(1 - \gamma_c)^{1.07}\right]\bigg/\left[1 + \left\{1.04\left(\frac{1 - \gamma_c}{1 + \gamma_c}\right)^{1.32}\right\}^{4.5}\right]^{0.222}(1 + \gamma_c)^{0.25} \ .$$

Therefore, since

$$\gamma_c = \frac{D_{RT}}{D_{OTR}} = \frac{39}{250} = 0.156 \quad ,$$

$$f_0 = 6.6$$

and thus

$$C = 6.6(78)^{0.78}(19)^{0.75}(39)^{1.07} \times 9.81 \, N = 888250 \, N \quad .$$

(i) Central eccentric B10 fatigue life[2] can be obtained from the equation

$$L_h = \frac{10^6}{60N_0}\left(\frac{C}{W}\right)^{\frac{10}{3}} h$$

$$= \frac{10^6 \times 20}{60 \times 5000}\left(\frac{888250}{252500}\right)^{\frac{10}{3}} = 4400 \, h \quad .$$

(ii) Similarly the outer eccentric $L_h = 44000$ h. Therefore, the overall B10 fatigue life of the triple eccentric roller tooth/inner cam ring/outer tooth ring load bearing faces is

$$L_h = 3700 \, h \quad .$$

C.4   Eccentric cam rolling element bearing life

It can be deduced from Fig 10 that at high power, such as considered in this design study, the B10 fatigue life is negligible and consequently rolling element bearings are unsuitable.

C.5   Roller tooth oscillatory plain hydrodynamic bearing design

(a)   Central eccentric roller tooth bearing:

The central eccentric maximum roller tooth load, $T = RF_{max} \sin \alpha$, therefore

$$T = 7.5 \times 10^4 \times 0.42 = 3.15 \times 10^4 \, N \quad ,$$

roller tooth bearing length $L_0 = 39$ mm
roller tooth diameter $D_{RT} = 39$ mm .
roller tooth speed $N_{RT} = (D_{OTR}/D_{RT})(N_I/G) = 3150$ rev/min.

From Ref 9, Fig 6 the non-dimensional load factor

$$W' = \frac{W}{\eta_e N L_0 D} \left(\frac{C_0}{D}\right)^2$$

$$= \frac{3.15 \times 10^4 (10^{-3})^2}{10^{-2} \left(\frac{3150}{60}\right) 39 \times 10^{-3} \times 39 \times 10^{-3}} = 39$$

where C_0/D =   the ratio of diametral clearance to bearing diameter = 10^{-3}

   η_e =   effective dynamic viscosity = 10^{-2} N s m^{-2} for D Eng RD 2497 oil at 70°C

   W =   T = bearing load

   N = N_{RT}

and D = D_{RT} .

From Ref 9, Fig 6, the minimum film thickness

$$h_0 = 0.065(10^{-3})39 \times 10^{-3} = 2.5 \ \mu m$$

indicating the lower regions of thin film lubrication.

The bearing eccentricity

$$\varepsilon = 0.87 \quad .$$

Wilcock's criterion[15] for the critical speed at which turbulence, and thus excessive friction loss, begins is

$$N_C = \frac{1560\nu}{D^{\frac{1}{2}} C_0^{\frac{3}{2}}} \ rev/min$$

$$= \frac{1560 \times 11 \times 10^{-6}}{(39 \times 10^{-3})^{\frac{1}{2}} \times (39 \times 10^{-6})^{\frac{3}{2}}} = 356770 \ rev/min$$

where ν =   kinematic viscosity = 11×10^{-6} m^2/s for D Eng RD 2497 oil at 70°C.

The roller tooth speed N_{RT} is well below that at which turbulence begins.

Consider the bearing to have an axial groove of length b = 0.8 L_0 and width v = 0.4 L_0.

From Ref 9, page 36, for the above groove dimensions Q_p' = 0.85.

For feed pressure $p_f = 4.1 \times 10^5$ Pa (60 psia) and from Ref 9, Fig 14, the pressure induced flow rate

$$Q_p = p_f h_f^3 Q_p' / \eta_i$$

$$= \frac{4.1 \times 10^5}{3 \times 10^{-2}} (39 \times 10^{-6})^3 \times 0.82 \times 0.85 \; m^3/s = 0.57 \; cm^3/s$$

where h_f = maximum film thickness in bearing,

η_i = dynamic viscosity at inlet to bearing = 3×10^{-2} N s m^{-2} for D Eng RD 2497 oil at 40°C.

From Ref 9, Fig 8, the theoretical flow rate

$$Q_E = L_0 C_0 D N Q_E'$$

$$= (39 \times 10^{-3})(39 \times 10^{-6})(39 \times 10^{-3}) \left(\frac{3150}{60} \right) (1.18) \; m^3/s$$

$$= 3.6 \; cm^3/s \quad .$$

From Ref 9, Fig 10, the velocity induced flow rate

$$Q_v = 0.8 L_0 C_0 D N Q_v'$$

$$= 0.8(39 \times 10^{-3})(39 \times 10^{-6})(39 \times 10^{-3}) \left(\frac{3150}{60} \right) (1.06) \; m^3/s$$

$$= 2.6 \; cm^3/s \quad .$$

From Ref 9, Fig 16, the friction power loss

$$H_f = H' \eta_e N^2 L_0 D^2 \left[\frac{D}{C_0} \right] \quad ,$$

$$= 128(10^{-2}) \left(\frac{3150}{60} \right)^2 (39 \times 10^{-3})(39 \times 10^{-3})^2 (10^3)$$

$$= 209 \; W \quad .$$

Pumping power loss $H_p = p_f \times Q_p = 4.1 \times 10^5 \times 0.57 \times 10^{-6} = 0.23$ W .

For axial groove bearings, temperature is given by

$$\theta_e - \theta_i = \left[\frac{K}{C\rho}\right]\left[\frac{H_f}{Q_E}\right]$$

when

$$(Q_p + Q_v) > Q_E \left\{ \begin{array}{l} \text{Substitute } Q_E \text{ for } (Q_p + Q_v) \\ \text{if } (Q_p + Q_v) < Q_E \end{array} \right\}$$

where   K   =   power loss factor usually assumed to be 0.8

C   =   heat capacity of lubricant J/kg $^\circ$C

ρ =   density of lubricant kg/m^3

$\left. \begin{array}{l} \\ \\ \end{array} \right\}$ $C\rho \doteq 1.7 \times 10^6$ for hydrocarbon oils.

θ_e =   effective temperature of lubricant in loaded part of bearing, $^\circ$C

θ_i =   lubricant inlet temperature, $^\circ$C.

Therefore

$$\theta_e - \theta_i = \frac{0.8}{1.7 \times 10^6}\left[\frac{209}{3.17 \times 10^{-6}}\right] = 31^\circ \quad .$$

The maximum temperature θ_{max} in the bearing can be estimated approximately from:

$$\theta_{max} - \theta_i = 2(\theta_e - \theta_i)$$

therefore

$$\theta_{max} = 2 \times 31 + 40 = 102^\circ C \quad .$$

Maximum acceptable temperatures for hydrocarbon oils can be as high as 120°C and therefore θ_{max} is acceptable in this bearing design.

(b)   Outer eccentric roller tooth bearing:

The outer eccentric maximum roller tooth load, T = 1.57 × 10^4 N. Adopting the bearing design method of Ref 9 illustrated above and using the same values of N_{RT}, D_{RT}, L_0, η_i, η_e, c_0, p_f, b and w , the following values are obtained:

W' = 19.7	Q_E = 3.5 cm^3/s
ϵ = 0.78	Q_v = 2.3 cm^3/s
h_0 = 4.3 µm	H_f = 155 W
N_C = 356770 rev/min	H_p = 0.2 W
Q_p = 0.49 cm^3/s	$\theta_e - \theta_i$ = 26°C
	θ_{max} = 92°C .

The bearing design is acceptable. It is apparent that the assumption : central eccentric roller tooth bearing loss = sum of outer eccentric roller tooth bearing losses, is not strictly correct in this case since the latter are about 50% higher than the former. However, this is of little consequence since the sum total of the roller tooth oscillatory hydrodynamic bearing losses is small. Total oscillatory bearing power loss,

$$H_{OJT} = [209.23 + 2(155.2)]n_{RT} \ W$$

$$= 9873 \ W$$

where $n_{RT} = 19 =$ number of roller teeth per eccentric cam.

The total oscillatory hydrodynamic bearing power loss percentage = 0.66%.

C.6   Eccentric cam plain hydrodynamic bearing design

(a)   Central cam bearing:

The central cam bearing load, $W = 25.25 \times 10^4$ N (see section C.3). Using the design method of Ref 9 the following bearing features result:

Bearing speed	N	5000 rev/min
Bearing diameter	D	100 mm
Bearing length	L_0	100 mm
Diametral clearance to bearing diameter ratio	C_0/D	1.2×10^{-3}
Effective dynamic viscosity of D Eng RD 2497 oil at 70°C	η_e	10^{-2} N s m^{-2}
Non-dimensional load factor	W'	44
Bearing eccentricity	ε	0.88
Minimum film thickness	h_0	7.2 μm
Critical speed	N_C	41742 rev/min
Dynamic viscosity at inlet to bearing for D Eng RD 2497 oil at 40°C	η_i	3×10^{-2} N s m^{-2}
Feed pressure	P_f	4.1×10^5 Pa
Axial groove length	b	$0.8 \ L_0$
Axial groove width	w	$0.4 \ L_0$
Pressure induced flow rate	Q_P	16.7 cm^3/s
Theoretical flow rate	Q_E	119 cm^3/s
Velocity induced flow rate	Q_v	86 cm^3/s
Friction power loss	H_f	7670 W
Pumping power loss	H_P	6.8 W
	$\theta_e - \theta_i$	35°C
	θ_{max}	110°C

The bearing design is acceptable.

(b) Outer cam bearings:

The outer cam bearing load, $W = 12.62 \times 10^4$ N. Using the design method of Ref 9, the following bearing features result:

N	=	5000 rev/min		
D	=	100 mm		
L_0	=	70 mm		
C_0/D	=	1.2×10^{-3}		
n_e	=	10^{-2} N s m^{-2}		
n_i	=	3×10^{-2} N s m^{-2}		
W'	=	32		
ϵ	=	0.88		
h_0	=	7.2 μm		
N_C	=	41317 rev/min		

P_f	≈	4.1×10^5 Pa
b	=	$0.8 L_0$
w	=	$0.4 L_0$
Q_p	=	20.7 cm^3/s
Q_E	=	92.4 cm^3/s
Q_v	=	68 cm^3/s
H_f	=	4992 W
H_p	=	8.5 W
$\theta_e - \theta_i$	=	27°C
θ_{max}	=	94°C

The bearing design is acceptable.

As in section C.5, the assumption: central cam bearing loss = sum of the losses in the two outer cam bearings is incorrect but this is of little conse-quence since the losses are small.

The total power loss in the three cam bearings is 17678 W which is equiva-lent to a percentage power loss of 1.2%.

C.7 Friction power losses in the roller tooth/eccentric cam/outer ring contact zone

The losses due to sliding between roller tooth, eccentric cam and outer ring can be estimated taking the minimum film thickness h_0 from Dyson and Wilson[16], and available experimental data[7] as guides to the type of lubrication and friction coefficient to be expected.

From section C.3, the maximum Hertzian stress of the central cam's roller teeth is S = 1340 MPa, and that of the outer cam's roller teeth is S = 950 MPa. The Hertzian stress used in the experimental work of Ref 7 was S = 1200 MPa, and is fairly representative of the present situation.

Now, $D_{RT} = 39$ mm, $L_{RT} = 78$ mm, and $N_{RT} = 3150$ rev/min

$$N_{IC} = \frac{D_{OTR}}{D_{ICR}} \frac{N_I}{G} = \frac{500}{461} \frac{5000}{20} = 270 \text{ rev/min} \quad .$$

Thus, the eccentric cam peripheral speed,

$$U_1 = \frac{\pi}{60} D_{IC} N_{IC} = 6.5 \text{ m/s}$$

and if sliding does not occur between the roller tooth and the eccentric cam the
roller tooth peripheral speed $U_2 = 6.5$ m/s. If no sliding occurs between the
roller tooth and the outer ring, the peripheral speed of the latter is also
6.5 m/s.

However, if sliding occurs, the sliding speed $V_s = U_1 - U_2$, and from
Ref 7, for s = 1200 MPa and a temperature of 363 K, conditions likely in the
roller tooth contact zone, $V_s = 0.4$ m/s and $\bar{U} = 0.5 V_s = 0.2$ m/s.

The load per unit length acting between the central sets most heavily
loaded roller tooth and the outer ring is

$$W_0 = \frac{RF_{max}}{L_{RT}} = \frac{7.5 \times 10^4}{78 \times 10^{-3}} = 96 \times 10^4 \text{ N/m} \quad .$$

Similarly, $W_0 = 48 \times 10^4$ N/m for the most heavily loaded roller tooth of the
outer sets.

From Dyson and Wilson[16], the minimum film thickness between heavily loaded
cylinders is

$$h_0 = 1.6R(\alpha_1 E)^{0.6} \left[\frac{\eta(0,\theta)\bar{u}}{ER}\right]^{0.7} \left(\frac{W_0}{ER}\right)^{-0.13} \quad .$$

For dynamic viscosity, $\qquad\qquad\qquad\qquad\qquad \eta = 7 \times 10^{-3}$ N s/m^2
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (for D Eng RD 2497 oil at
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ about 90°C)

$\qquad$ pressure coefficient of viscosity, $\qquad\quad \alpha_1 = 1.74 \times 10^{-8}$ m^2/N

$\qquad$ roller tooth radius, $\qquad\qquad\qquad\qquad R_{RT} = 19.5$ mm

$\qquad$ modulus of elasticity of steel, $\qquad\quad E = 21 \times 10^4$ MPa

$$h_0 = 1.6 \times 19.5 \times 10^{-3}(1.74 \times 10^{-8} \times 21 \times 10^{10})^{0.6} \left[\frac{7 \times 10^{-3} \times 0.2}{19.5 \times 10^{-3} \times 21 \times 10^{10}}\right]^{0.7}$$

$$\times \left[\frac{96 \times 10^4}{21 \times 10^{10} \times 19.5 \times 10^{-3}}\right]^{-0.13}$$

= 0.024 um $\qquad$ for the central cam's most heavily loaded roller tooth, and is

0.026 um $\qquad$ for the outer cam's most heavily loaded roller tooth.

Consider the same sliding speed to occur between the roller teeth and eccentric cams

$$W_0 = \frac{RF_{max} \cos \alpha}{L_{RT}} = \frac{7.5 \times 10^4 \times 0.91}{78 \times 10^{-3}} \approx 87 \times 10^4 \quad N/m$$

for the most heavily loaded central eccentric cam roller tooth,

$$W_0 = 43.5 \times 10^4 \quad N/m$$

for the most heavily loaded outer eccentric cam roller tooth. Thus

$$h_0 = 0.023 \quad \mu m$$

for the most heavily loaded central eccentric cam roller tooth,

$$h_0 = 0.025 \quad \mu m$$

for the most heavily loaded outer eccentric cam roller tooth.

  Sliding friction power loss

$$H_s = FV_s \quad W$$

where F = frictional force = coefficient of friction (μ) $\times$ normal load.
All the above values of h_0 suggest boundary or mixed film lubrication.

  It can be seen from Ref 7 (Fig 13) that the 363 K temperature curves at rolling speeds of 3.9 m/s and 6.6 m/s would yield traction coefficient values less than $\mu = 0.04$ at a sliding speed $V_s = 0.42$ m/s for Hertzian stress $S = 1200$ MPa and lubricant dynamic viscosity 8.5×10^{-3} N s/m^2.

  Thus, choosing $\mu = 0.02$ for conservative estimates,

(a) Central eccentric cam roller tooth power loss:

  (i) between roller tooth and outer ring is

$$H_{RTOR_s} = 0.02 \times 7.5 \times 10^4 \times 0.4 \quad W$$
$$= 600 \quad W \quad ;$$

(ii) between roller tooth and eccentric cam is

$$H_{RTEC_S} = 0.02 \times 7.5 \times 10^4 \times 0.91 \times 0.4 \ W$$
$$= 540 \ W \quad .$$

(b) Outer eccentric cam roller tooth power loss:

(i) between roller tooth and outer ring is

$$H_{RTOR_S} = 0.02 \times 3.75 \times 10^4 \times 0.4 \ W$$
$$= 300 \ W \quad ;$$

(ii) between roller tooth and eccentric cam is

$$H_{RTEC_S} = 0.02 \times 3.75 \times 10^4 \times 0.91 \times 0.4 \ W$$
$$= 270 \ W \quad .$$

Since there are 19 roller teeth per eccentric cam the total system frictional power loss

$$_TH_S = \left[(600 + 540) + 2(300 + 270)\right]19 = 43300 \ W \quad .$$

This is essentially a pessimistic figure since strictly speaking only up to 50% of the roller teeth transmit the power at any instant, and the most heavily loaded roller tooth was chosen to assess the sliding frictional losses. However it is difficult to provide an accurate value of μ and so the above calculated power loss will be retained. Consequently the percentage power loss due to roller tooth sliding is considered to be no greater than 2.9%.

C.8 Disc friction loss

Stepanoff[17] gives the disc friction loss (both sides) as

$$H_d = KD^2 \sigma V^3 \quad .$$

For this design exercise

$$D_{ICR} = 461 \ mm = 1.8 \ ft \quad \text{and} \quad D_{OTR} = 600 \ mm = 2.0 \ ft \quad ,$$

$$v = \pi \times 1.8 \times \frac{250}{60} \times \frac{D_{OTR}}{D_{ICR}} = 26 \text{ ft/s} \quad (7.9 \text{ m/s})$$

$$\nu = 12 \times 10^{-5} \text{ ft}^2/\text{s} \quad (11 \times 10^{-6} \text{ m}^2/\text{s})$$

$$\sigma_{OIL} = 50 \text{ lb/ft}^3 \quad (760 \text{ kg/m}^3)$$

$$R_e = \frac{26 \times 1.8}{2 \times 12 \times 10^{-5}} = 1.8 \times 10^5 \quad .$$

Let the spacing between the eccentric cams and the casing be $0.1D_{ICR}$. From Pfleiderer's curves[18], $K = 5.5 \times 10^{-8}$, therefore

$$H_d = 5.5 \times 10^{-8} \times (1.8)^2 \times 50 \times 26^3 \text{ hp}$$

$$= 114 \text{ W per eccentric} \quad .$$

Total disc friction loss $H_{d_T} \simeq 342$ W, ie a negligible percentage power loss.

C.9 Design summary

In theory, a follower tooth reduction gear unit can be designed to have an efficiency of approximately 95.5% and an acceptable roller tooth B10 fatigue life. The eccentric cam bearings should be of hydrodynamic/hydrostatic form with precautionary surface film additives and hydraulic jacking to avoid excessive wear on 'start-up' and 'shut-down'. This also applies to the oscillatory roller tooth bearings. The theoretical performance indicated presumes that the modifications suggested to reduce friction are in themselves satisfactory and that main bearing losses are small.

Figs 15 and 16 show the basic design of the gearbox. Component materials:

Gearcase	- magnesium alloy RZS (density = 1.81 g/cm³)
Outer rings	- steel (7.9 g/cm³)
Main bearings, input shaft, roller teeth and torque reacting faces of the cage	- steel
Remainder of cage and output shaft	- titanium-manganese (4.5 g/cm³)
Eccentric cams	- titanium and steel
Rolamite strip	- Sandvik 11R51 stainless steel.

Power to weight ratio:

(1)   of the basic gearbox excluding an extra output shaft bearing (shown dotted in Fig 15) to react rotor bending moments is

$$5.5 \quad kW/kg \quad ;$$

(2)   of the basic gearbox with the extra bearing is

$$5.3 \quad kW/kg \quad ;$$

(3)   of the basic gearbox with the extra bearing and a right-angled drive input stage is

$$4.1 \quad kW/kg \quad .$$

Table 1

DIMENSIONAL LIMITATIONS OF THE FOLLOWER TOOTH GEAR

(a) $\rho_0 \simeq \rho_s = R_{RT} = 2.5a$

OTR PCD (m)	Geometric quantities dimensions (mm)	Gear ratio, G					
		100	80	60	40	20	10
		$\rho_s = R_{RT} = 2.5a$					
0.6	R	156a	127a	95a	64a	32a	16a
	a	1.9	2.3	3.1	4.5	8.7	16.2
	R_{RT}	4.75	5.8	7.7	11.2	21.6	40.5
	λ_{OTR}	19	24	31.5	48	96	192
	ρ_1	9.5	11.6	15.4	22.4	43.2	81
	$-\rho_2$	0	0	0	0	0	0
0.4	R	156a	127a	95a	64a	32a	16a
	a	1.2	1.6	2.05	3.0	5.8	10.8
	R_{RT}	3.0	3.9	5.1	7.5	14.4	27
	λ_{OTR}	12.7	16	21	32	64	128
	ρ_1	6.0	7.9	10.2	15.0	28.8	54
	$-\rho_2$	0	0	0	0	0	0
0.2	R	156a	127a	95a	64a	32a	16a
	a	0.6	0.8	1.0	1.5	2.9	5.4
	R_{RT}	1.5	1.95	2.6	3.75	7.2	13.5
	λ_{OTR}	6.4	8	10.5	16	32	64
	ρ_1	3.0	4.0	5.1	7.5	14.4	27
	$-\rho_2$	0	0	0	0	0	0
0.1	R	156a	127a	95a	64a	32a	16a
	a	0.3	0.4	0.5	0.75	1.45	2.7
	R_{RT}	0.75	1.0	1.3	1.9	3.6	6.7
	λ_{OTR}	3.2	4	5.3	8	16	32
	ρ_1	1.5	2.0	2.6	3.75	7.2	13.5
	$-\rho_2$	0	0	0	0	0	0

Table 1 (continued)

(b) $R_{RT} = R_{RTC}$, determined from equation (A-12)

OTR PCD (m)	Geometric quantities dimensions (mm)	Gear ratio, G					
		100	80	60	40	20	10
0.6	R	156a	128a	95a	64a	32a	15.8a
	a	1.9	2.3	3.1	4.5	8.7	16.2
	R_{RT}	4.75	5.9	7.8	11.7	23.5	47
	λ_{OTR}	19	24	31.5	48	96	192
	ρ_1	9.5	11.8	15.4	23	45	88
	$-\rho_2$	0	0	0	0	0	0
0.4	R	164a	122a	95a	64a	32a	15.8a
	a	1.2	1.6	2.05	3.0	5.8	10.8
	R_{RT}	3.1	4	5.2	7.8	15.8	30.67
	λ_{OTR}	12.7	16	21	32	64	128
	ρ_1	6.3	7.8	10.2	15.3	30	58
	$-\rho_2$	0.1	0	0	0	0	0
0.2	R	164a	124a	95a	64a	32a	15.8a
	a	0.6	0.8	1.0	1.5	2.9	5.4
	R_{RT}	1.55	2	2.6	3.9	7.9	15.33
	λ_{OTR}	6.4	8	10.5	16	32	64
	ρ_1	3.2	3.9	5.1	7.6	15	29
	$-\rho_2$	0.06	0	0	0	0	0
0.1	R	164a	124a	95a	64a	32a	15.8a
	a	0.3	0.4	0.5	0.75	1.45	2.7
	R_{RT}	0.75	1.0	1.3	2	3.9	7.66
	λ_{OTR}	3.2	4	5.3	8	16	32
	ρ_1	1.6	2.0	2.6	3.8	7.5	14.4
	$-\rho_2$	0.03	0	0	0	0	0

Table 1 (concluded)

(c) $\rho_s = 4a$ and $R_{RT} = R_{RT_C}$ ($\rho_s > R_{RT} > 2.5a$)

OTR PCD (m)	Geometric quantities dimensions (mm)	Gear ratio, G					
		100	80	60	40	20	10
		$\rho_s = 4a$					
0.6	R	295 or 200a	296 / 160a	292 / 120a	288 / 80a	276 / 40a	253 / 20a
	a	1.48	1.74	2.4	3.6	6.9	12.6
	R_{RT}	4.75	5.9	7.8	11.7	23.5	47
	λ_{OTR}	19	24	31.5	48	96	192
	ρ_1	10.7	12.9	17.4	26.1	51.1	97.4
	$-\rho_2$	1.17	1.06	1.8	2.7	4.1	3.4
0.4	R	197 or 200a	196 / 160a	195 / 120a	192 / 80a	184 / 40a	169 / 20a
	a	0.99	1.22	1.62	2.4	4.6	8.4
	R_{RT}	3.1	4.0	5.2	7.8	15.8	30.67
	λ_{OTR}	12.7	16	21	32	64	128
	ρ_1	7.06	8.88	11.68	17.4	34.2	64.3
	$-\rho_2$	0.86	0.88	1.28	1.8	2.6	3.0
0.2	R	99 or 200a	98 / 160a	97 / 120a	96 / 80a	92 / 40a	85 / 20a
	a	0.5	0.61	0.81	1.2	2.3	4.2
	R_{RT}	1.55	2.0	2.6	3.9	7.9	15.33
	λ_{OTR}	6.4	8	10.5	16	32	64
	ρ_1	3.55	4.44	5.84	8.7	17.1	32.1
	$-\rho_2$	0.45	0.44	0.64	1.1	1.3	1.4
0.1	R	49 or 200a	49 / 160a	49 / 120a	48 / 80a	46 / 40a	42 / 20a
	a	0.25	0.3	0.41	0.6	1.15	2.1
	R_{RT}	0.75	1.0	1.3	2.0	3.9	7.66
	λ_{OTR}	3.2	4	5.3	8	16	32
	ρ_1	1.75	2.2	2.94	4.4	8.5	16
	$-\rho_2$	0.25	0.2	0.34	0.4	0.7	0.7

LIST OF SYMBOLS

C	basic dynamic capacity	N
C_0	diametral clearance	m
D	diameter	m
D_{ICR}	eccentric cam diameter	m
D_0	cage maximum outer diameter	m
D_{OTR}	pitch diameter of outer ring waveform	m
D_{RT}	roller tooth diameter	m
E	modulus of elasticity	Pa
F	friction force	N
G	gear ratio	-
G_0	torsional modulus of elasticity	Pa
H_d	disc friction loss	-
H, H_f, H_p, H_s	power loss, bearing friction power loss, pumping power loss, sliding friction power loss	W
ICR	eccentric cam	-
IF	inertia force	N
J	polar moment of area	m^4
K	numerical coefficient	-
L_h	B10 fatigue life	h
L_0	journal bearing length	m
L_{RT}	length of roller tooth touching ICR profile	m
N	rotational speed	rev/min
N_C	critical speed at which turbulence begins	rev/min
N_I	input speed	rev/min
N_0	output speed	rev/min
N_{RT}	roller tooth speed	rev/min
OTR	outer ring	-
P_r	radial force on roller tooth	N
P_T	torque force	N
P_{θ_1}	roller tooth component force due to P_T	N
P_{θ_2}	roller tooth component force due to IF	N

LIST OF SYMBOLS (continued)

Q_P	feed pressure induced flow rate	m^3/s
Q_E	theoretical flow rate in full width film at groove	m^3/s
Q_V	velocity induced flow rate	m^3/s
R	radius	m
Re	Reynolds number	-
R_{ICR}	eccentric cam radius	m
R_{OTR}	pitch circle radius of outer ring waveform	m
R_{RT}	roller tooth radius	m
R_{RT_C}	modified roller tooth radius	m
RF	reaction force	N
S	Hertzian stress or pressure	Pa
T	torque reaction force; temperature	N; ^oC
T_0	output torque	N m
U_1, U_2	peripheral speed	m/s
$\bar{U}$	$0.5(U_1 + U_2)$, $(U_1 > 0; U_1 + U_2 > 0)$	m/s
V_s	sliding speed	m/s
W	bearing load	N
W'	load factor	-
W_0	load per unit length	N/m
X,Y	equispaced intersection points of two waveforms of equal amplitude and similar characteristic equations	-
a	half amplitude of waveform	m
b	groove length	m
e	eccentricity; exponent 2.718	m
f	frequency	Hz
f_0	factor depending on bearing material and type	-
h_0	minimum film thickness	m
i	number of rows of rolling elements per cam or race	-
ℓ_0	cage length	m
m	mass	kg
n	integer	-

LIST OF SYMBOLS (concluded)

n_{RT}	number of roller teeth per eccentric cam	-
P_f	lubricant feed pressure	N/m^2
r	radius	m
r_0	radial thickness of cage	m
t_0	cage end plate thickness	m
v	peripheral velocity	m/s
y_0	radial distance from centre line to cage wall neutral axis	m
α	angle at a point on the outer waveform between the normal at that point to the waveform and a radial line passing through the point	rad; ^{o}(deg)
α_1	pressure coefficient of viscosity	m^2/N
α_2	contact angle	rad; ^{o}(deg)
γ	$(\pi/2 - \alpha)$	rad; ^{o}(deg)
γ_c	D_{RT}/D_{OTR}, the diameter ratio	-
ϵ	bearing eccentricity	m
η	dynamic viscosity	$N\ s/m^2$; cP
θ, θ_1	angular displacement	rad; ^{o}(deg)
θ_0	angular deflection per unit length	rad/m
λ	wavelength	m
μ	coefficient of friction (traction coefficient)	-
ν	kinematic viscosity	m^2/s
ρ	radius of curvature	
ρ_0, ρ_s	minimum radii of curvature of waveform traced by roller tooth centre	m
ρ_1	maximum radius of curvature of outer ring waveform	m
ρ_2	minimum radius of curvature of outer ring waveform	m
σ	density	kg/m^3
τ_0	shear stress	
ω	angular velocity	rad/s

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16	A. Dyson A.R. Wilson	Film thicknesses in elastohydrodynamic lubrication at high slide/roll ratios. Proc. I. Mech. E., Vol 183, Part 3P (1968-69)
17	A.J. Stepanoff	Centrifugal and axial flow pumps. 2nd Edition, J. Wiley & Sons Inc
18	C. Pfleiderer	Die Kreiselpumpen. Julius Springer, p 99, Berlin (1955)
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Fig 1
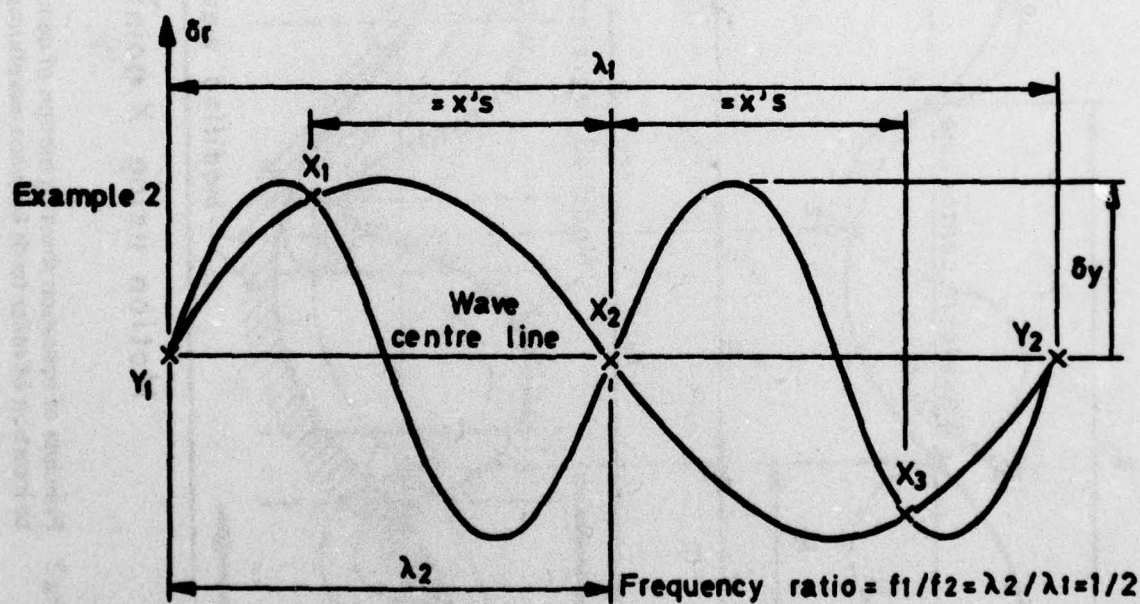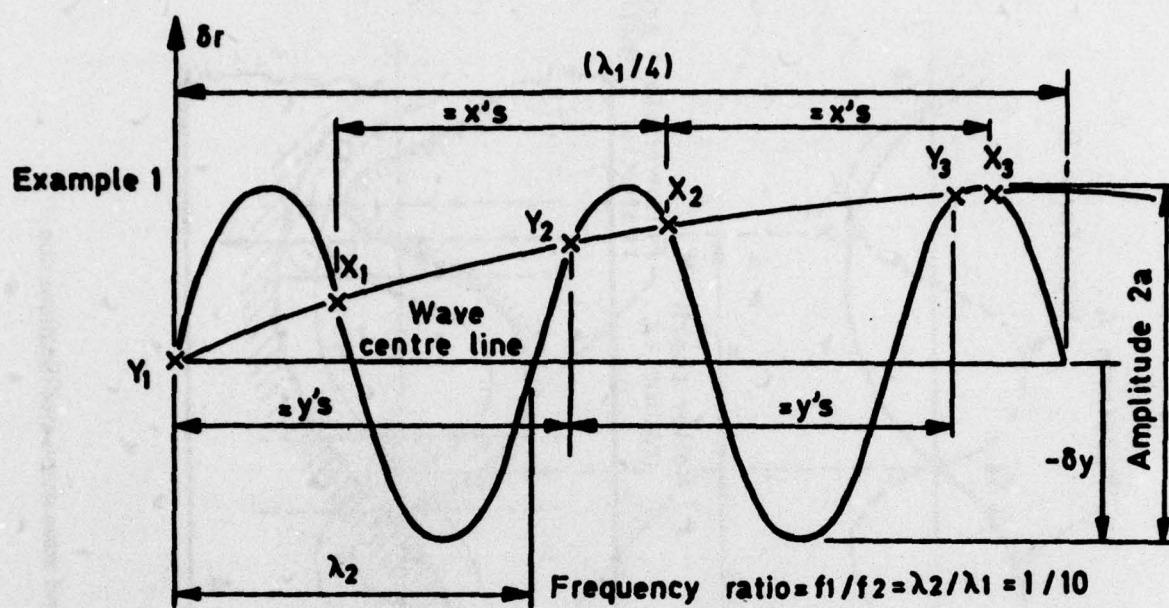
Fig 1 Sine-wave intersections at X and Y points
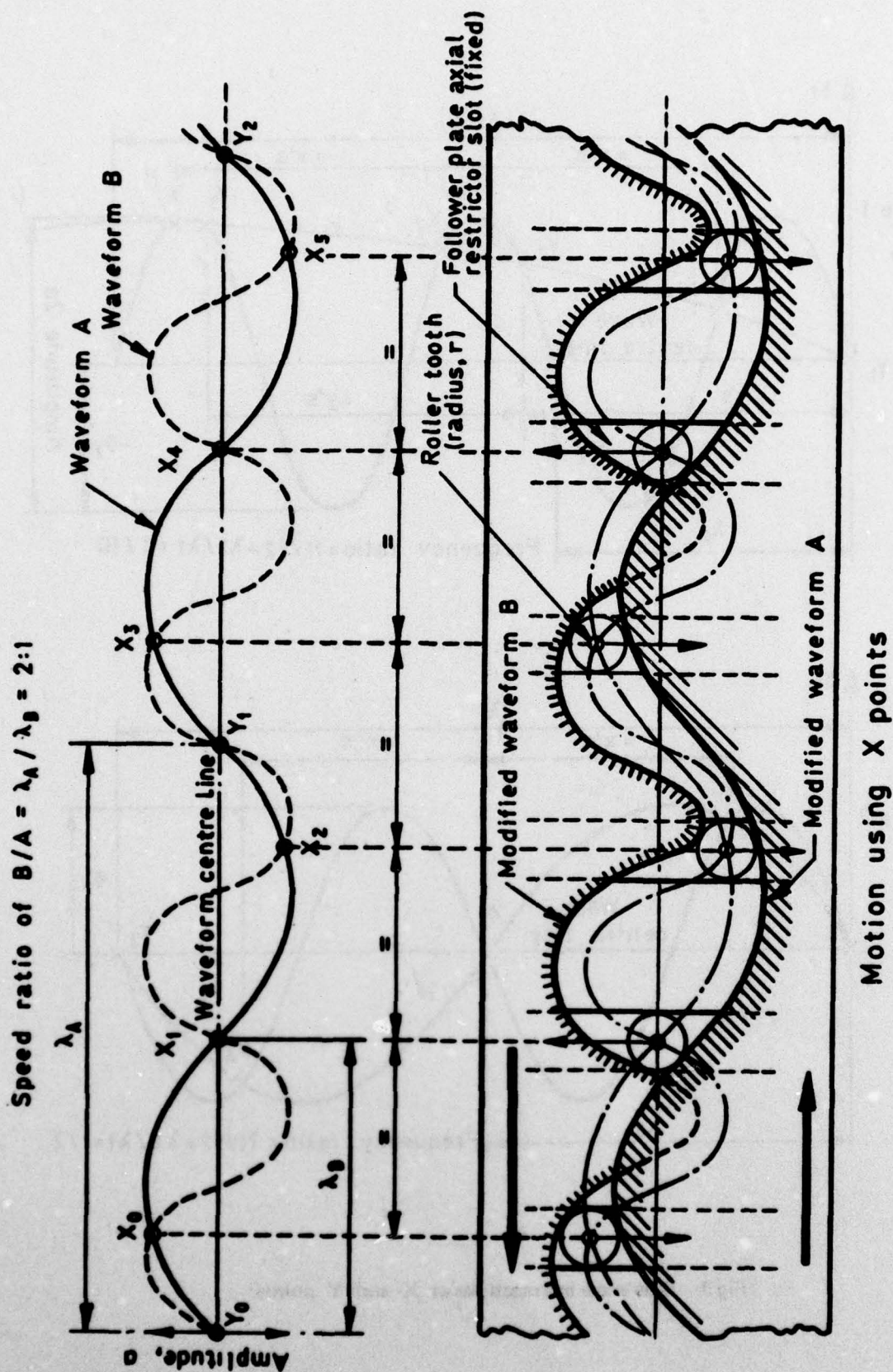
Fig 2   Principle of operation showing motion of components and wave shape modifications due to insertion of roller teeth between waveforms

Fig 3
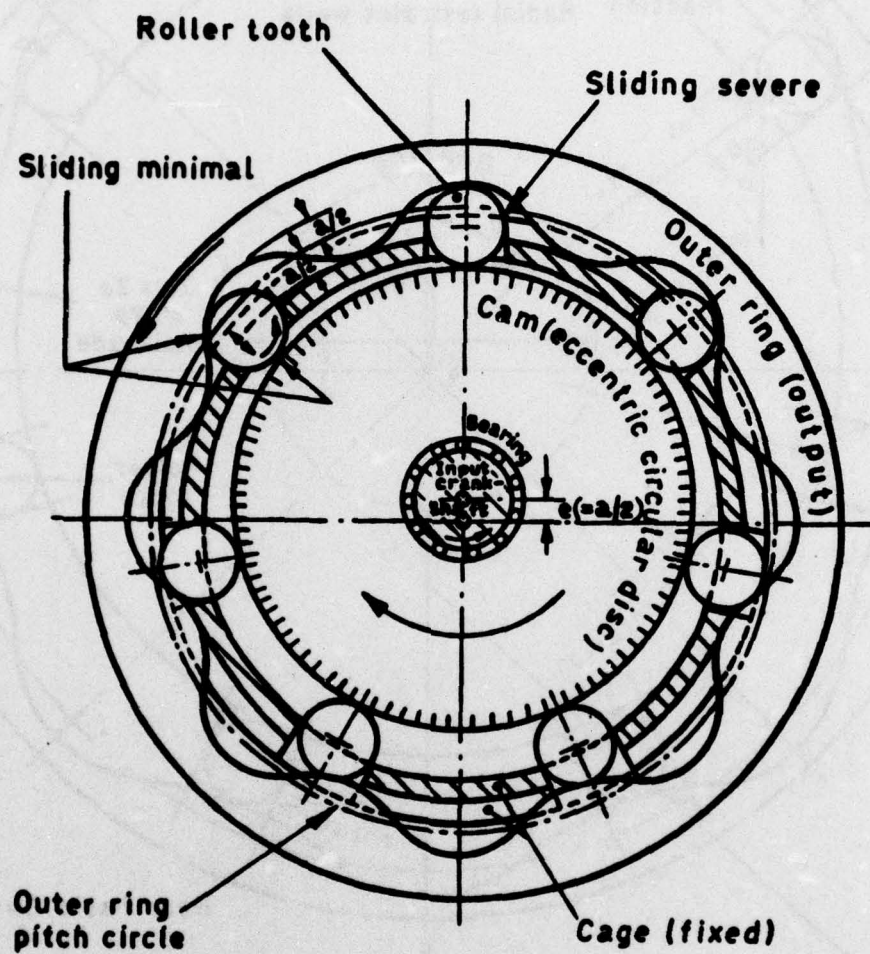
Fig 3   Simple cylindrical rotary design with speed ratio 8:1

Fig 4

a

Outer ring (fixed)

Torque reaction

Radial restraint walls

$P\theta_1$

$q\alpha$

Pr_1

$P\theta_2$

Bearing

θ

IF

2a = 2e wave amplitude

P_T

b

e

d

Pr_2

Roller tooth

Eccentric cam rotation

Torque reaction

Radial reaction

Cage rotation

Gear ratio G = 8:1

c
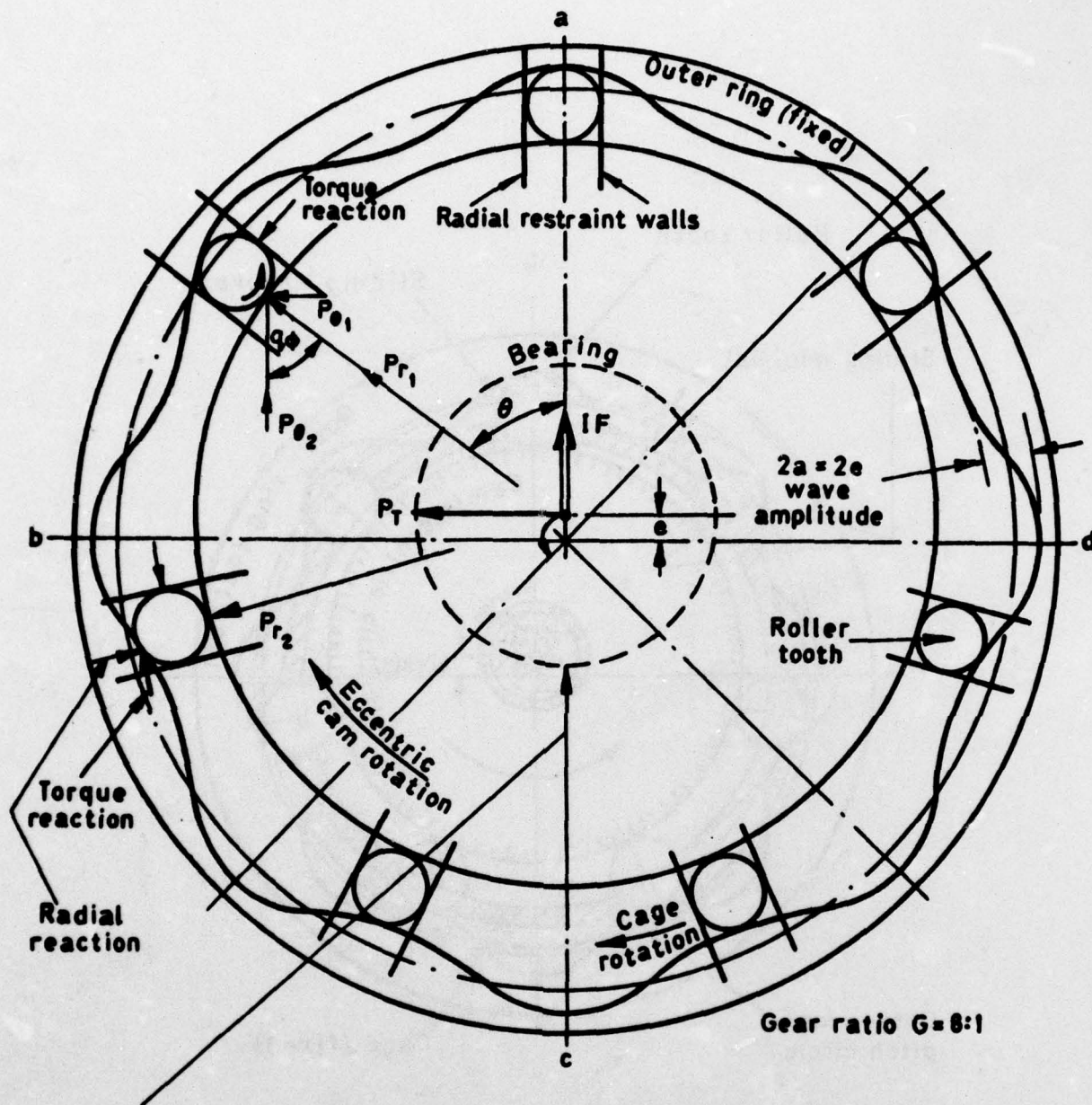
Reaction inertia force on bearing = 0 if the roller tooth zone has zero clearances, is pre-loaded, or the bearing clearances are greater than the total clearances between roller teeth, outer ring and eccentric cam

Fig 4   Loading and motion in a cylindrical rotary design

TR 77040
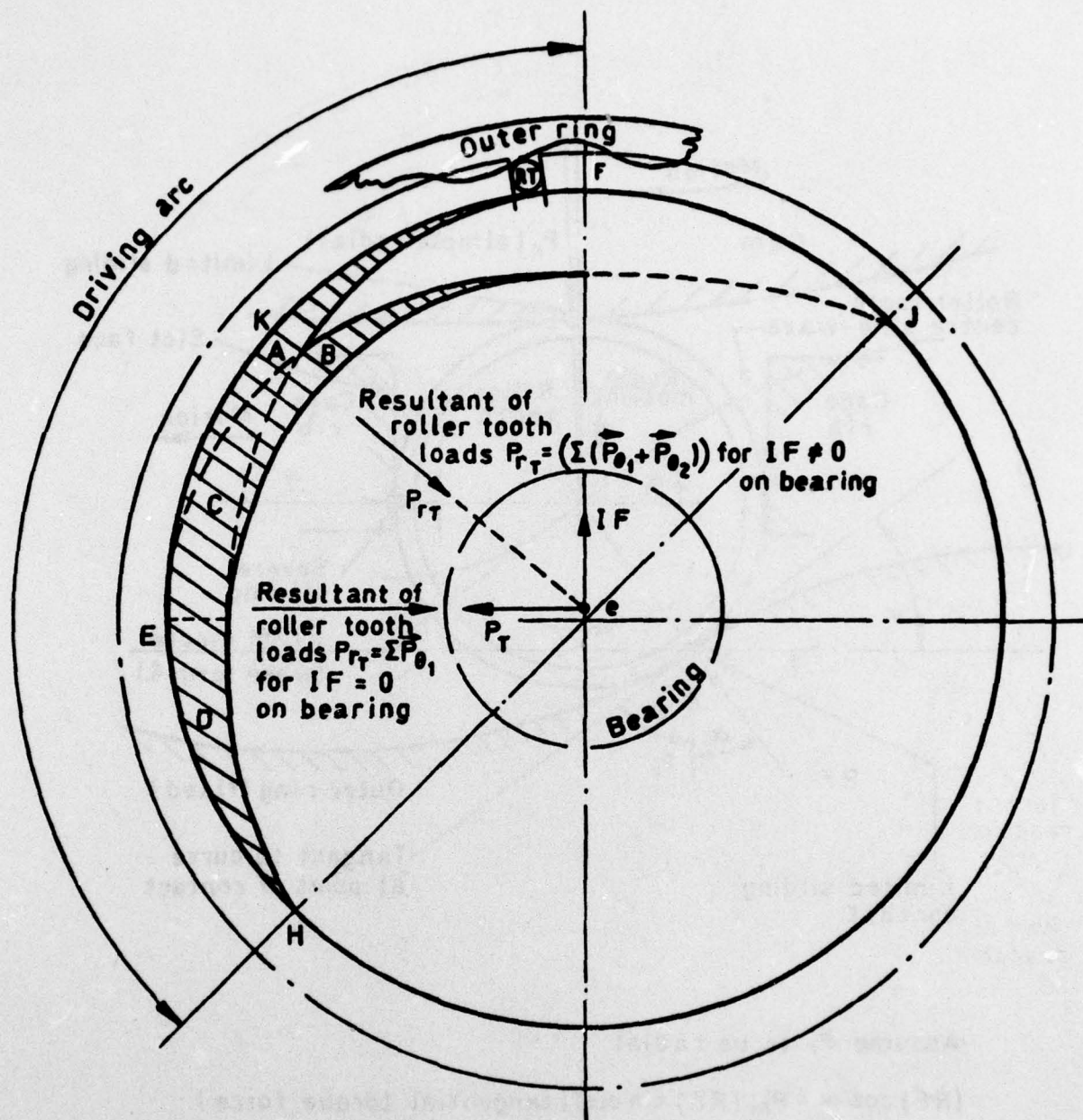
Fig 5

Assuming the cam bearing clearances are larger
than roller tooth clearances, the cam bearing
load is solely dependent on the torque reaction
radial load $(\Sigma \vec{P}_{\theta_1})$ neglecting friction

Fig 5   Loading on cam bearing at design speed

Fig 6

Assume P_r to be radial

$(RF)\cos \alpha = P_r$; $(RF)\sin \alpha = T$ (tangential torque force)

Largest normal force on roller tooth is RF

Fig 6 Loads on roller tooth and sliding between the roller tooth and the cage slot face
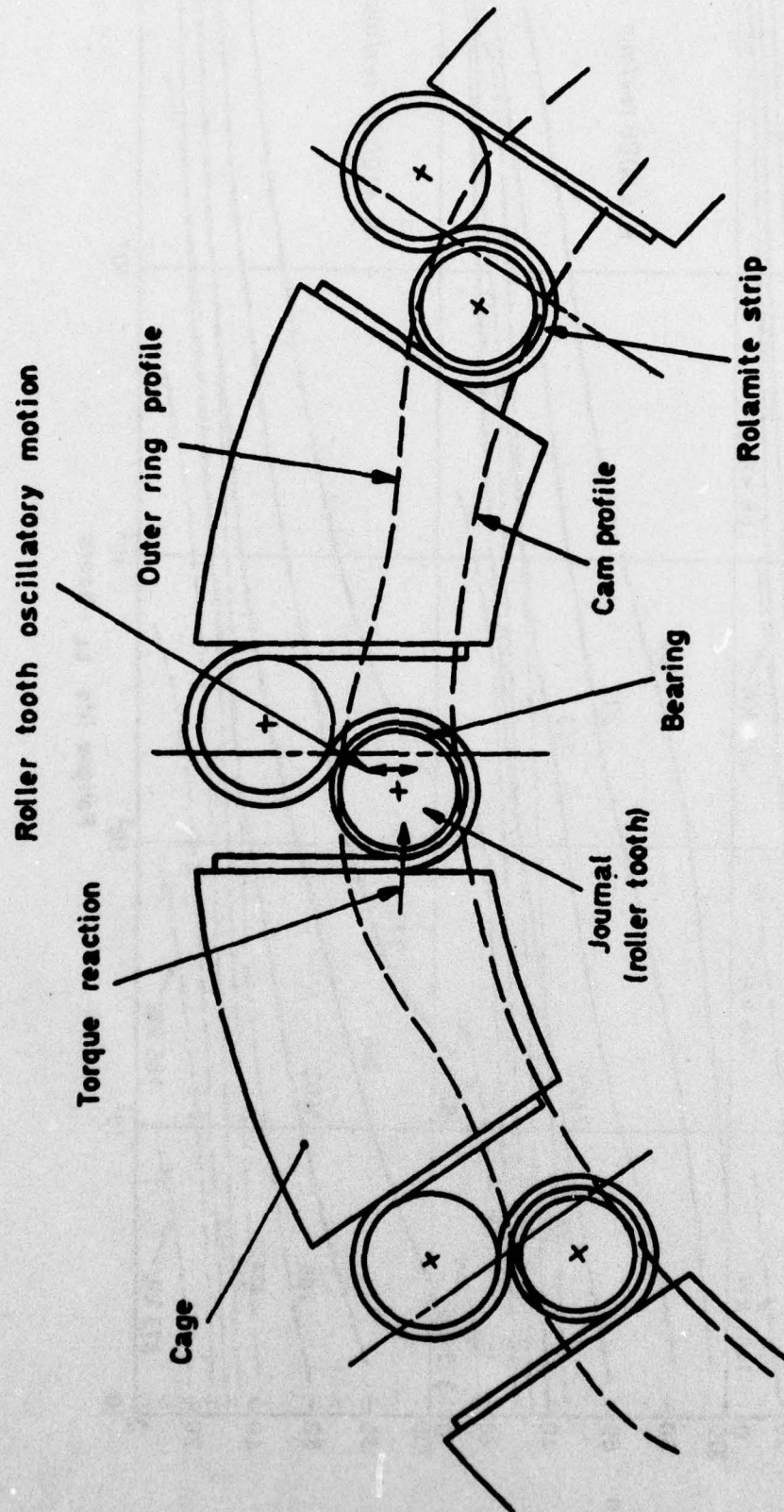
Fig 7

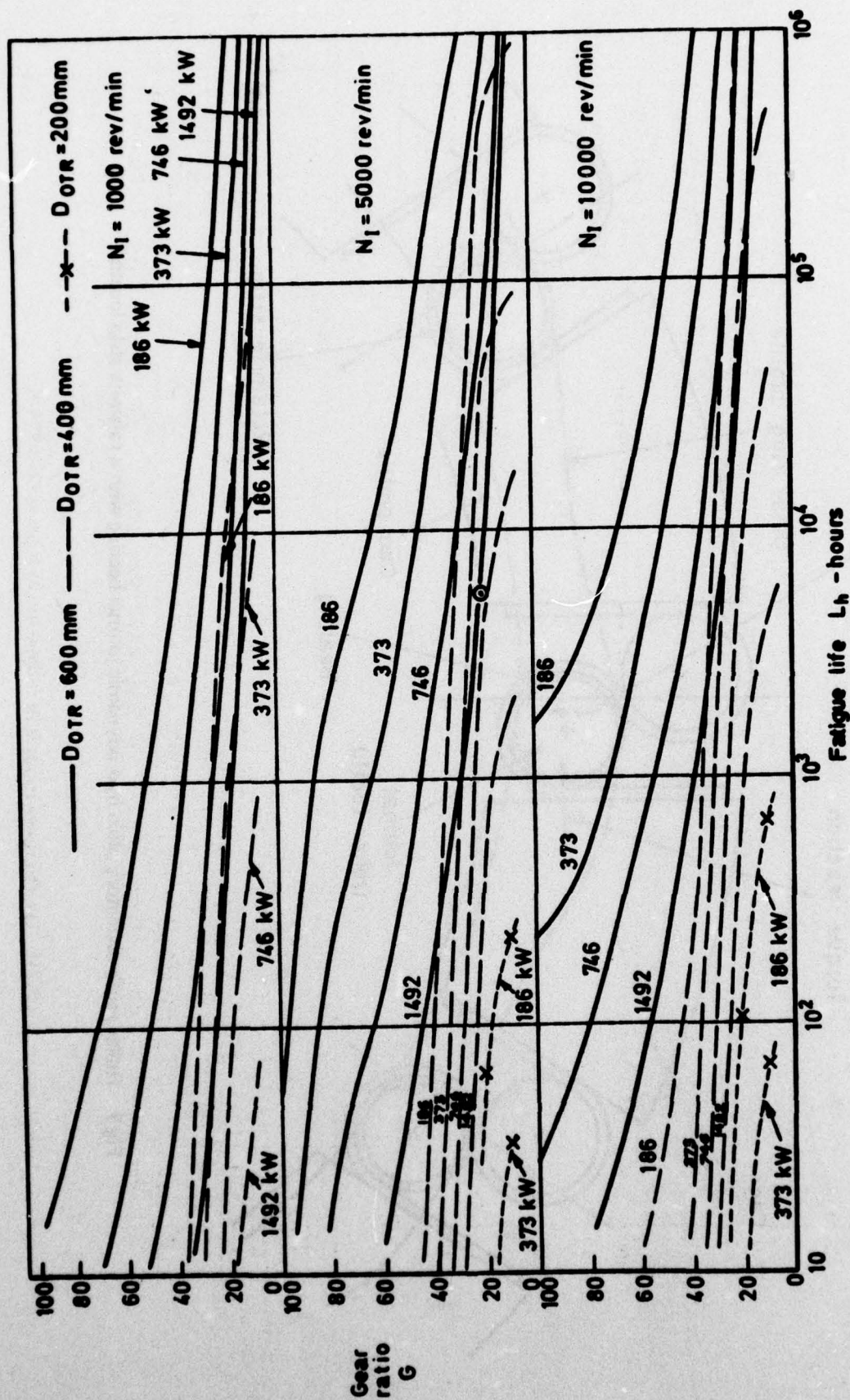Fig 7   Roller tooth oscillatory plain hydrodynamic journal bearing with a rolamite strip locator

Fig 8

Fig 8   Overall roller tooth B10 fatigue life in triple eccentric units

Fig 9

—— D_{ORR} = 600 mm ——— D_{ORR} = 400 mm -×--×- D_{ORR} = 200 mm

N_l = 20 000 rev/min

N_l = 30 000 rev/min

Fatigue life — L_h hours

Gear ratio G

For D_{ORR} = 200 mm

G	100	80	60	40	20	10
a mm	0.5	0.61	0.61	1.2	2.3	4.2
D_{RT} mm	3.1	4	5.2	7.8	15.8	30.67
L_{RT} mm	6.2	8	10.4	15.6	31.6	61.34
n_{RT}	99	79	59	39	19	9
$\alpha°$	26.5	25.2	26.2	26.5	26.5	26.5

For D_{ORR} = 400 mm

G	100	80	60	40	20	10
a mm	0.99	1.22	1.62	2.4	4.6	8.4
D_{RT} mm	6.2	8	10.4	15.6	31.6	61.34
L_{RT} mm	12.4	16	20.8	31.2	63.2	122.7
n_{RT}	99	79	59	39	19	9
$\alpha°$	26.5	25.2	26.2	26.5	26.5	26.5

Common design criteria — For D_{ORR} = 600 mm

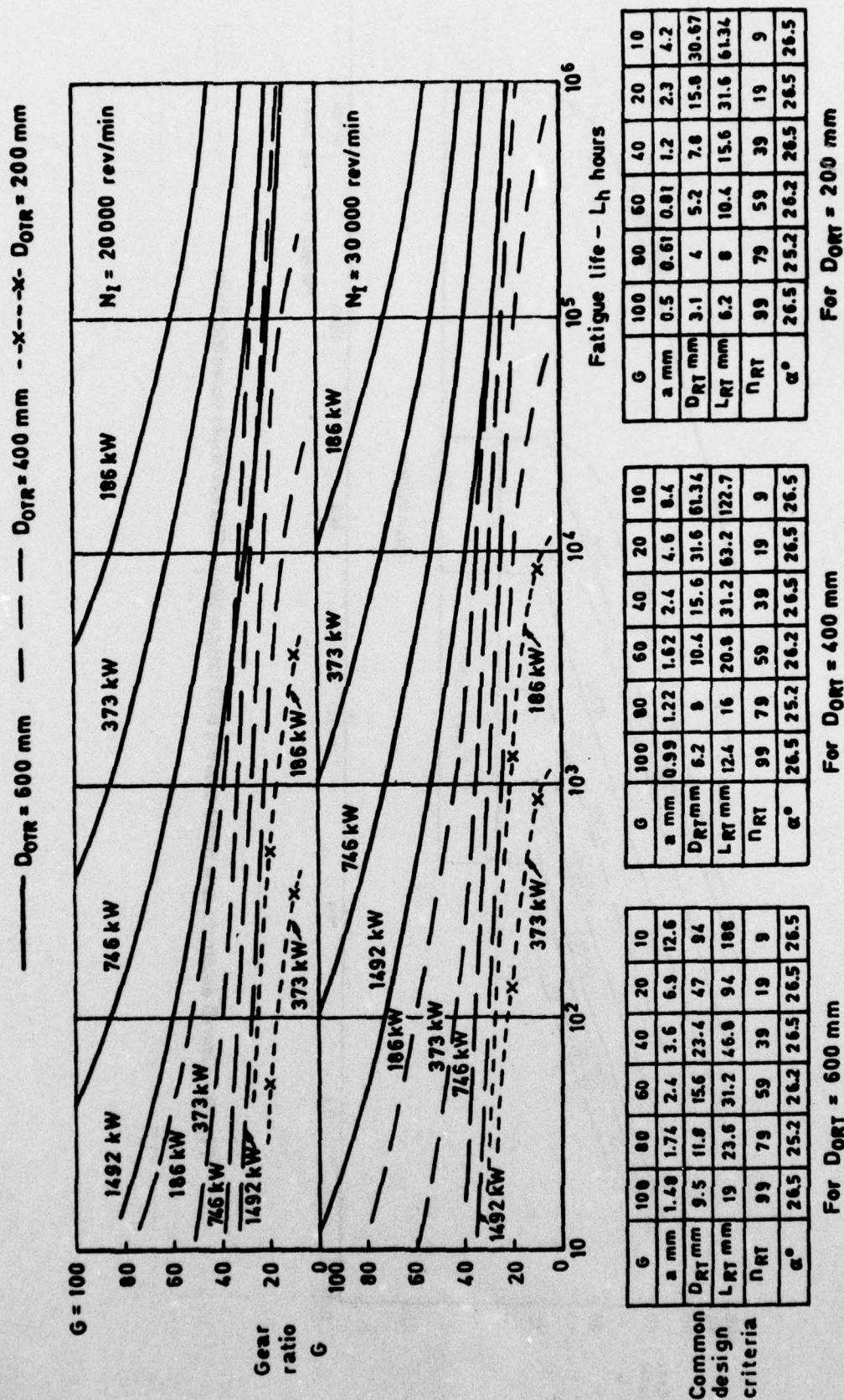| G	100	80	60	40	20	10
a mm	1.48	1.74	2.4	3.6	6.9	12.6
D_{RT} mm	9.5	11.8	15.6	23.4	47	94
L_{RT} mm	19	23.6	31.2	46.8	9.4	188
n_{RT}	99	79	59	39	19	9
$\alpha°$	26.5	25.2	26.2	26.5	26.5	26.5

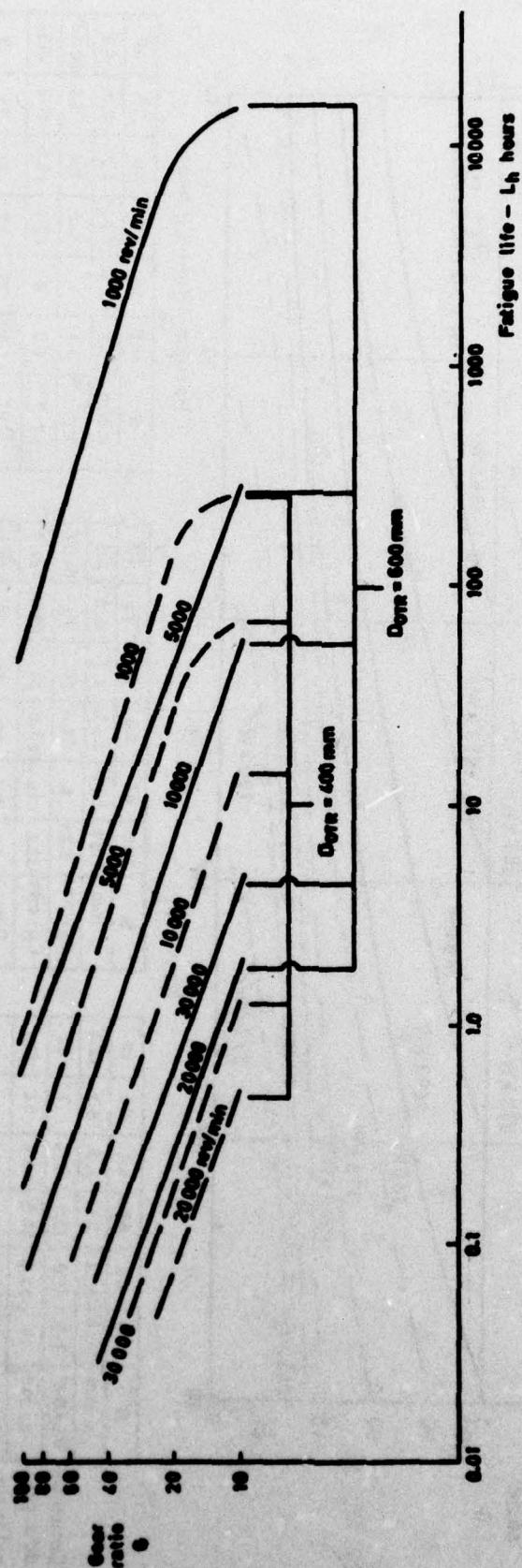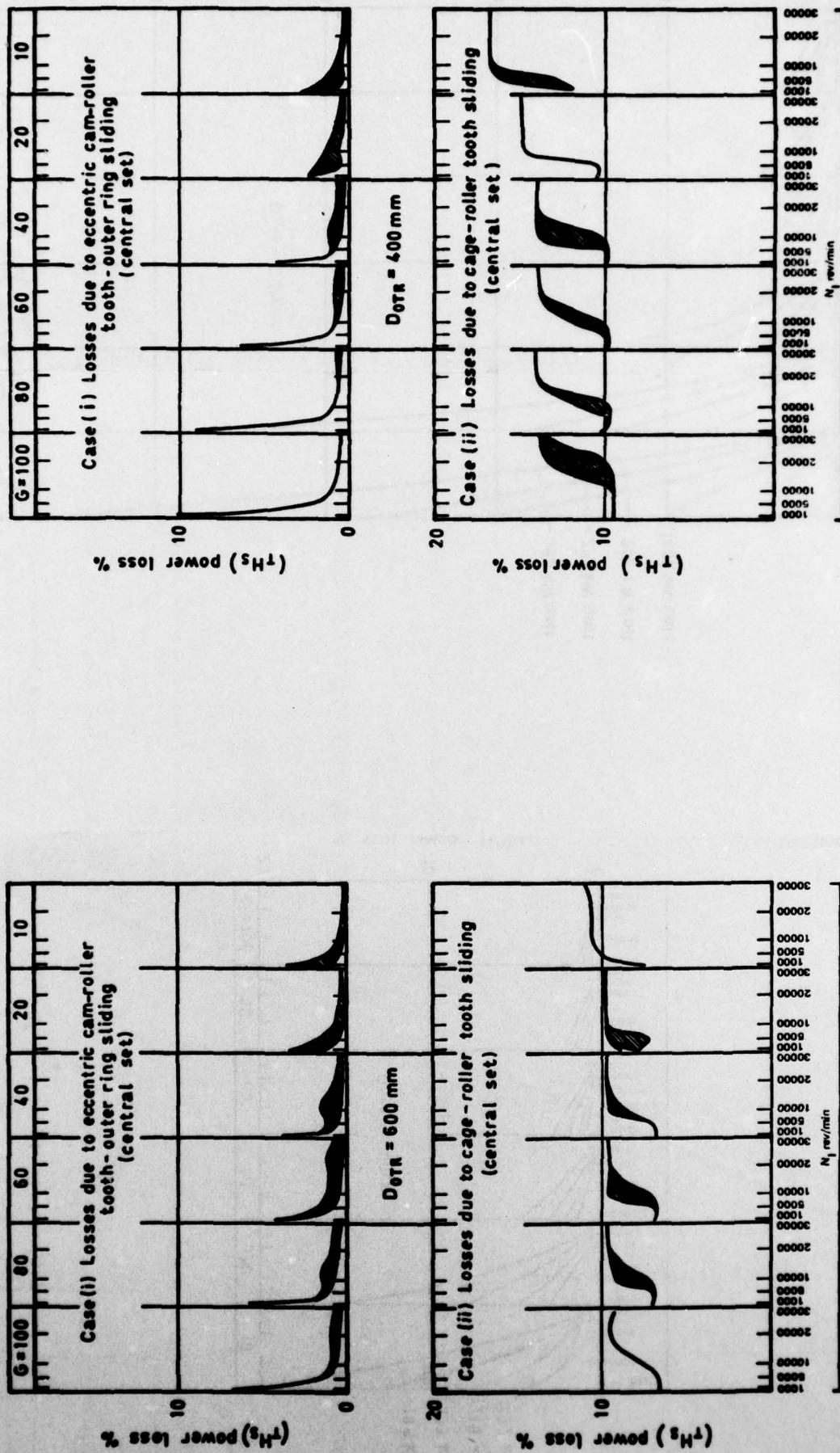Fig 9   Overall roller tooth B10 fatigue life in triple eccentric units

Fig.10

Fig 10   Overall eccentric cam roller bearing B10 fatigue life in 186 kW triple eccentric units

Fig 11

Total power loss = 2 x central set power loss
(ie total percentage loss as above)

The thick bands indicate the theoretical variation in losses that
arise as chosen friction coefficients change with change in
hertzian stress and minimum film thickness

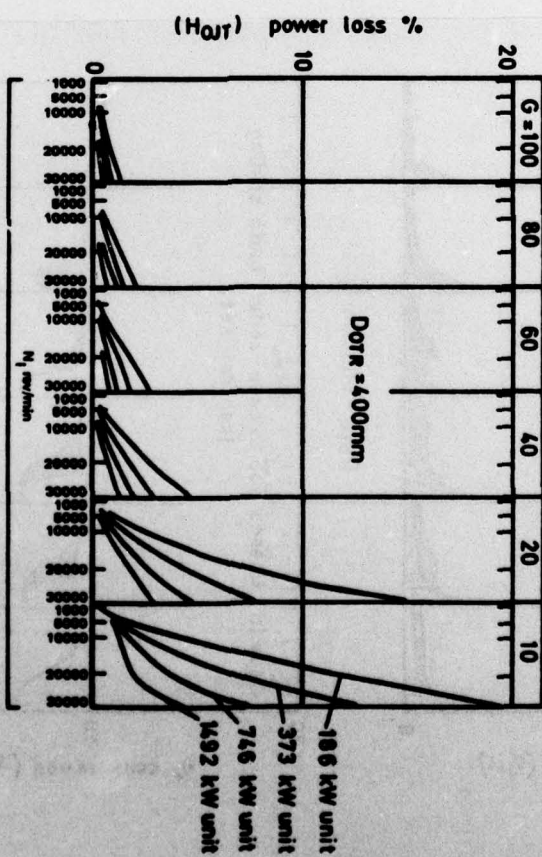Fig 11 Roller tooth sliding friction losses in the central sets of triple eccentric units
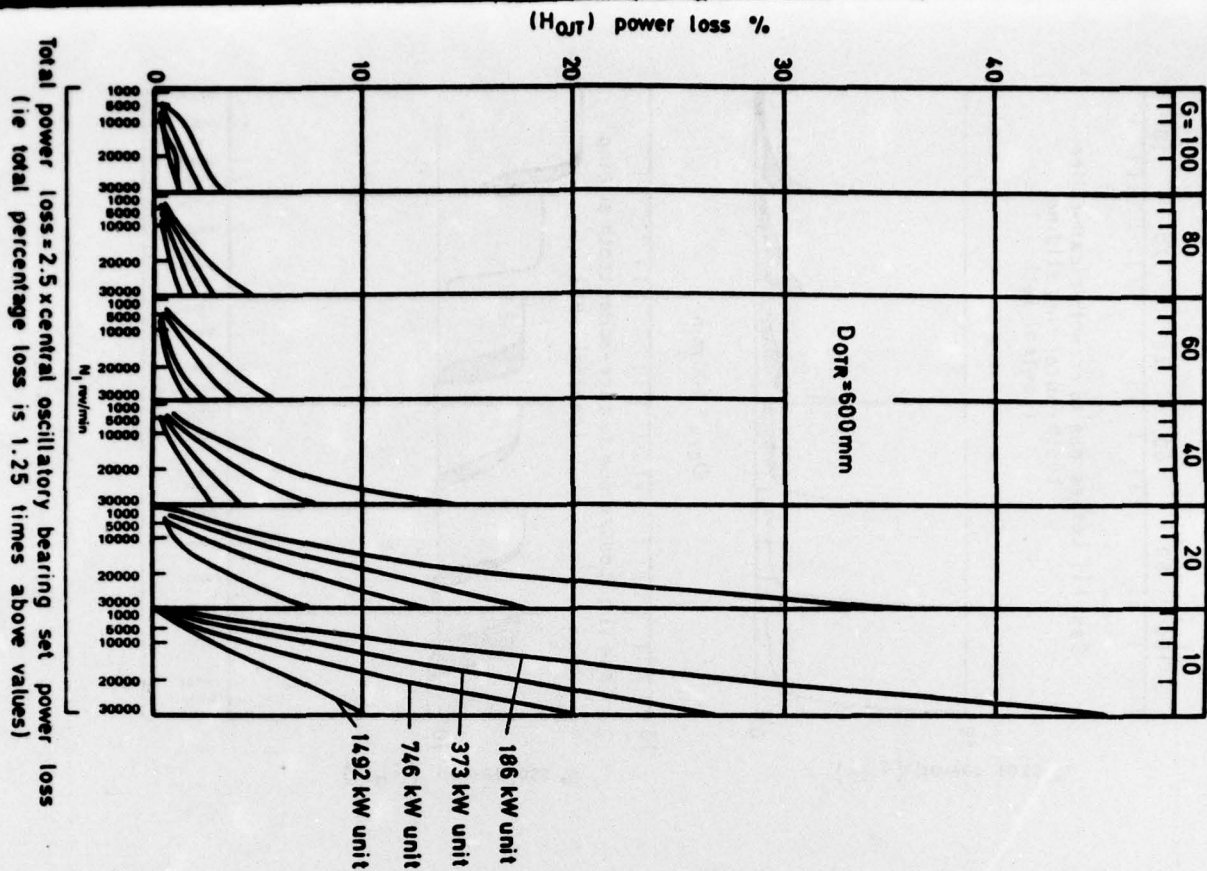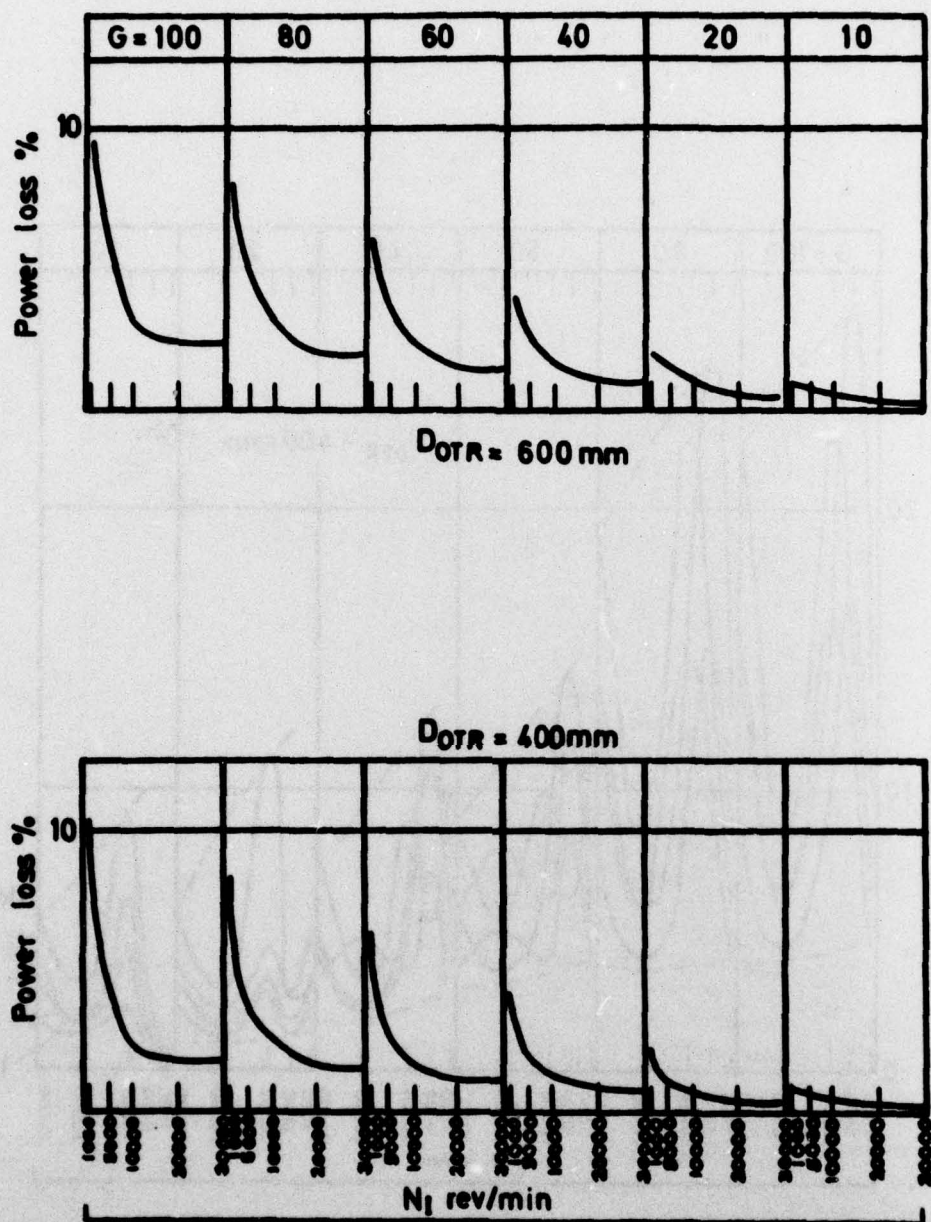
Fig 12

Fig 12   Roller tooth oscillatory hydrodynamic bearing losses in the central sets of triple eccentric units

Total power loss = 2.5 × central oscillatory bearing set power loss
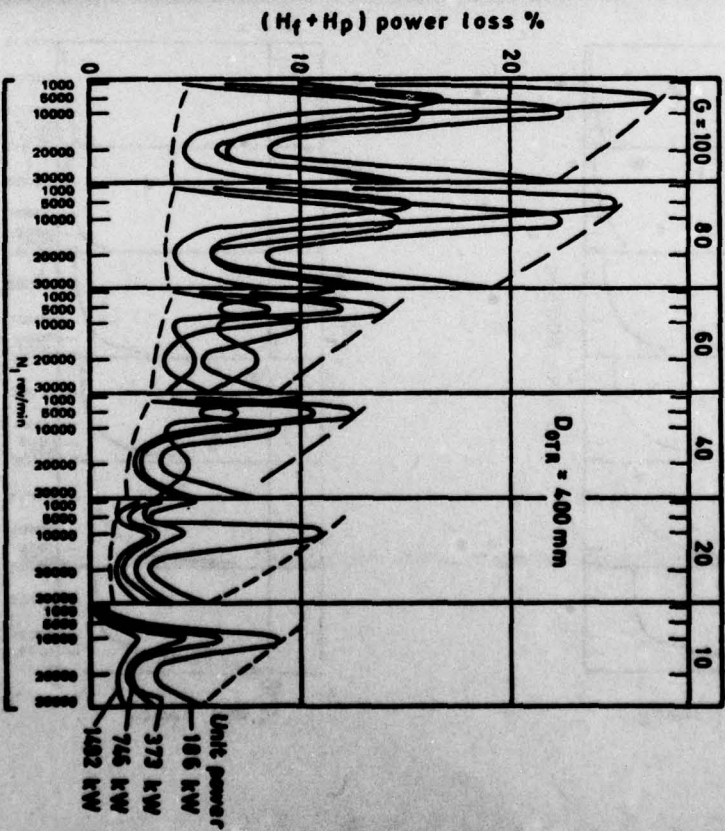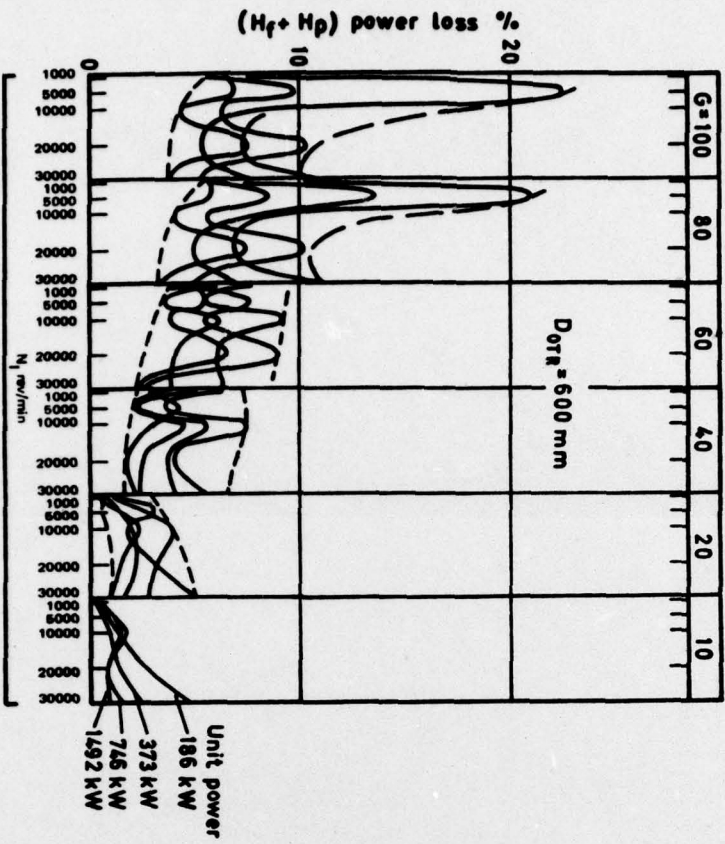(ie total percentage loss is 1.25 times above values)

Fig 13

Total power loss = 2 x central eccentric cam bearing power loss
(ie total percentage loss as above)

Fig 13   Central eccentric cam rolling bearing power loss for a 373 kW triple eccentric unit

Fig 14

(H_f + H_p) power loss %

$D_{OTR} = 600$ mm

Total power loss = 2.5 × central eccentric cam bearing power loss
(ie total percentage loss is 1.25 times above values)

The full lines curves within the dotted band lines indicate
the theoretical variation in losses that arise as bearing sizes
and parameters are varied

(H_f + H_p) power loss %

$D_{OTR} = 400$ mm

Fig 14   Central eccentric cam hydrodynamic bearing losses in triple eccentric units

Fig 15

ROLAMITE STRIPS

ROLLER
TOOTH

OSCILLATORY
BEARING

IDLER
CYLINDER

SECTION AA
ECCENTRIC CAM 3 ELEMENTS NOT SHOWN

SECTION BB

Input conditions — 1492 kW (2000 hp) at 5000 rev/min
Gear ratio — 20:1
(seals not shown)
RB = Roller bearing
BB = Ball bearing
Scale: ¾

OIL FEED ROUTE TO ROLLER TEETH

HELICOPTER MAIN SUPPORT BEARING
(SPHERICAL ROLLER THRUST BEARING
SKF 29328)

OUTPUT

(SUGGESTED ADDITIONAL BEARINGS REQUIRED
TO REACT ROTOR BENDING MOMENTS)

Note — The drawing is not meant to be fully detailed and some of the
unimportant feature sizes can be modified to increase overall
strength. Methods of manufacture and details of assembly are
not shown.

OSCILLATORY ROLLER
BEARING TOOTH

ECCENTRIC
CAM
3

ECCENTRIC
CAM
2

ECCENTRIC
CAM
1

OIL FEED ROUTE
TO ECCENTRIC CAM
BEARINGS

INPUT

Fig 15   Basic gearbox scheme for a free harmonic tooth reduction drive

Fig 16

Oil bleed seal

Cage

Bearing

Collar

Outer ring (fixed)

Eccentric cam

Roller tooth

Section A–A

Rolamite strip

Hydrodynamic bearing

Outer ring

Cage

Eccentric cam

Roller teeth

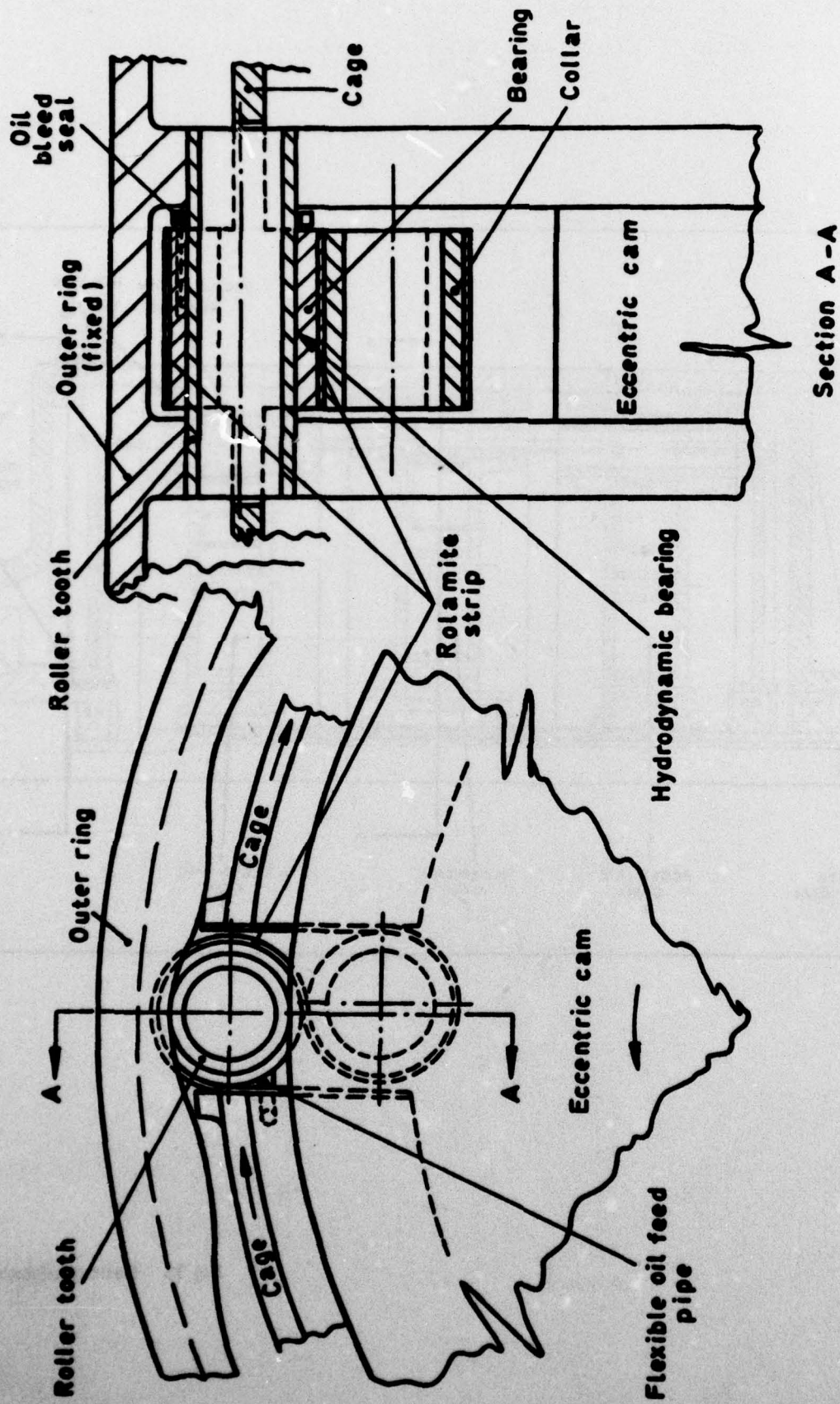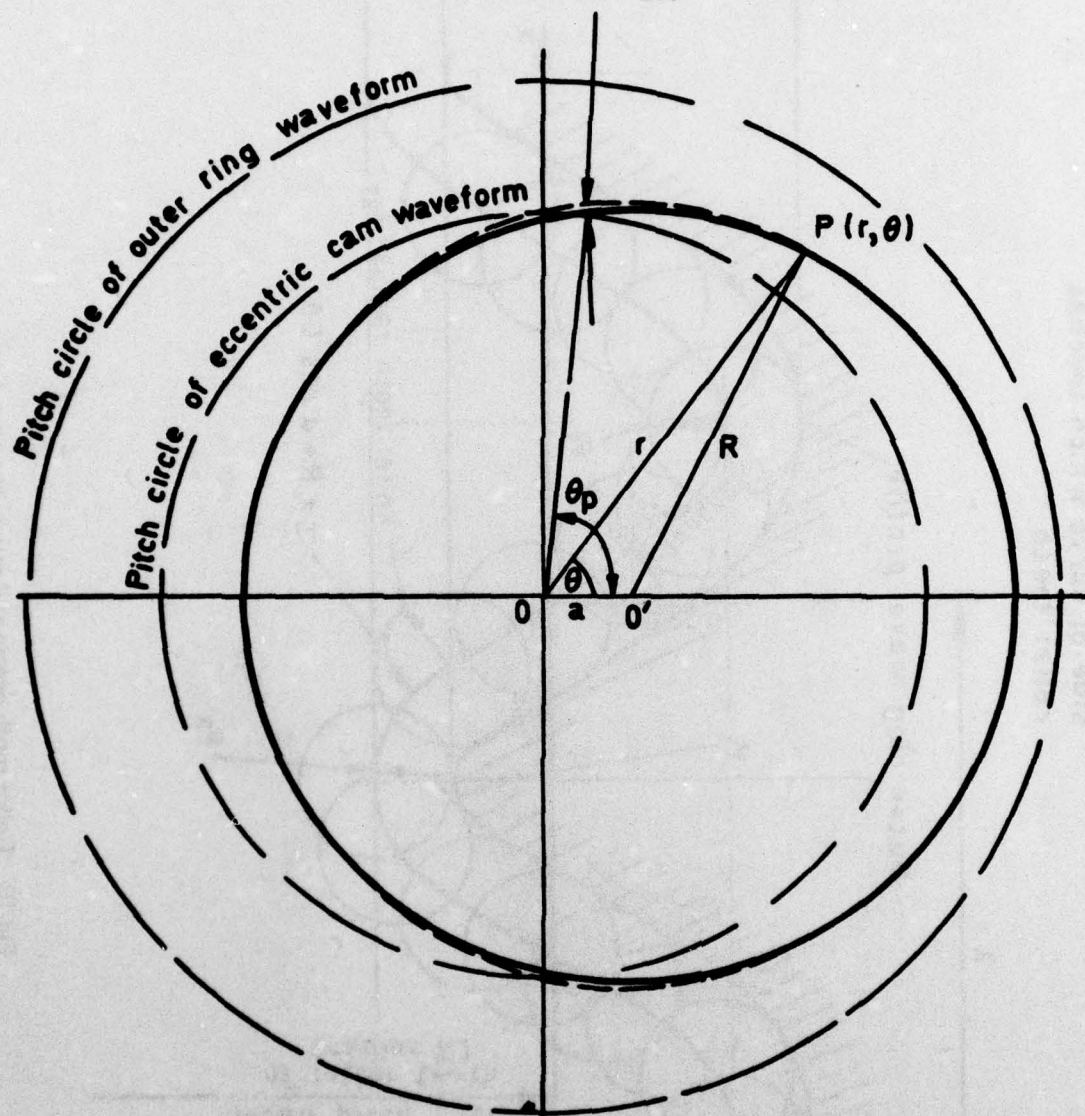Cage

Cage

A

A

Flexible oil feed pipe

Fig 16 Oil feed system in roller tooth-rolamite strip design

Fig 17

The error reduces as $\theta \rightarrow 0, \pi, 2\pi$

The short dotted line is the cosinusoidal curve and this blends with the circular profile as the error reduces

Error at $\theta p = \dfrac{a^2}{2R}(\sin^2 \theta p)$

O is the centre of the outer ring waveform pitch circle and the centre of the inner cam waveform pitch circle O' is the centre of the eccentric cam profile circle

Fig 17   Error diagram for the cylindrical rotary design

Fig 18
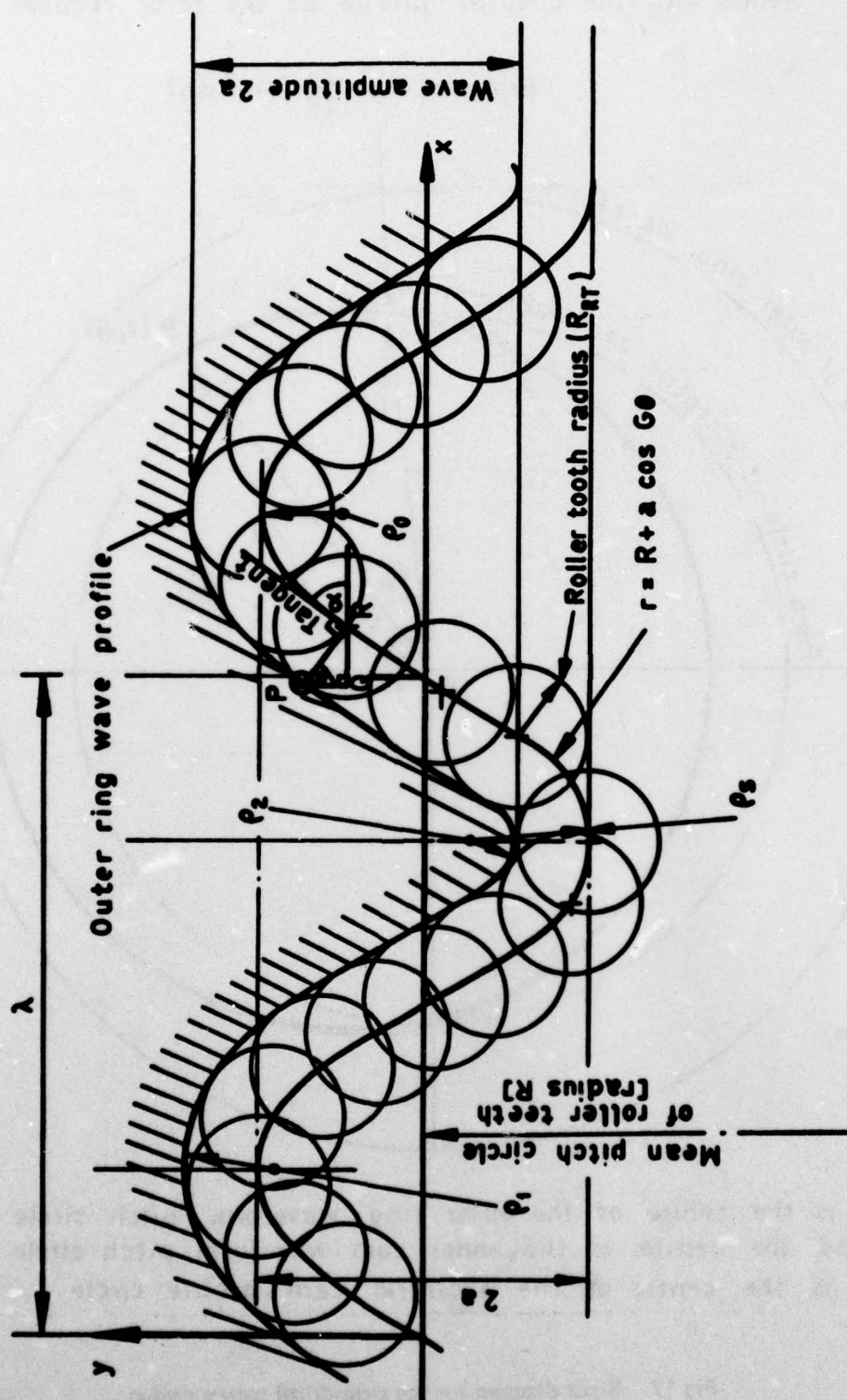
Fig 18 Roller tooth centre and outer ring wave traces

Fig 19

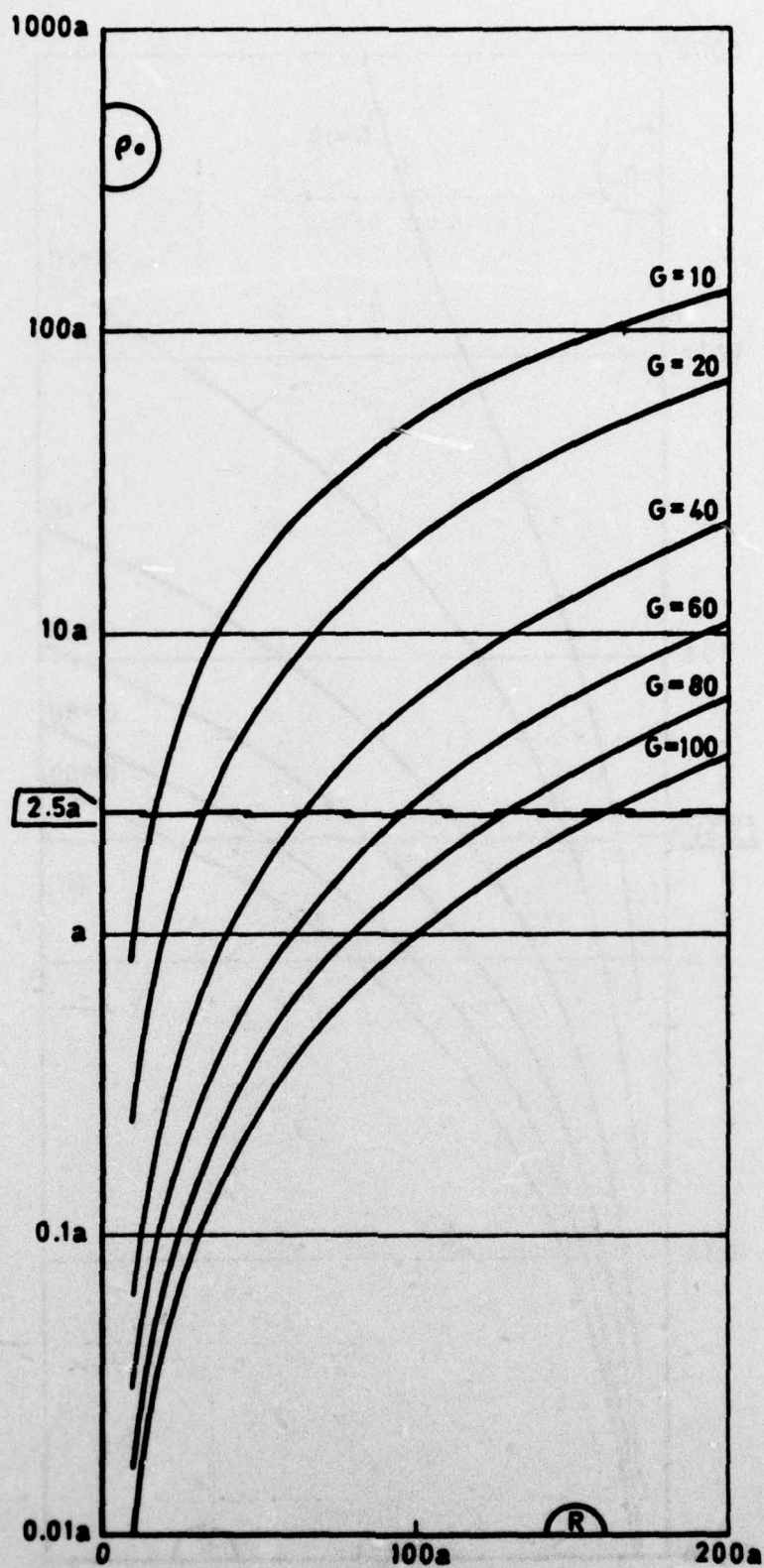
Fig 19   Relationship between radius of curvature and gear ratio

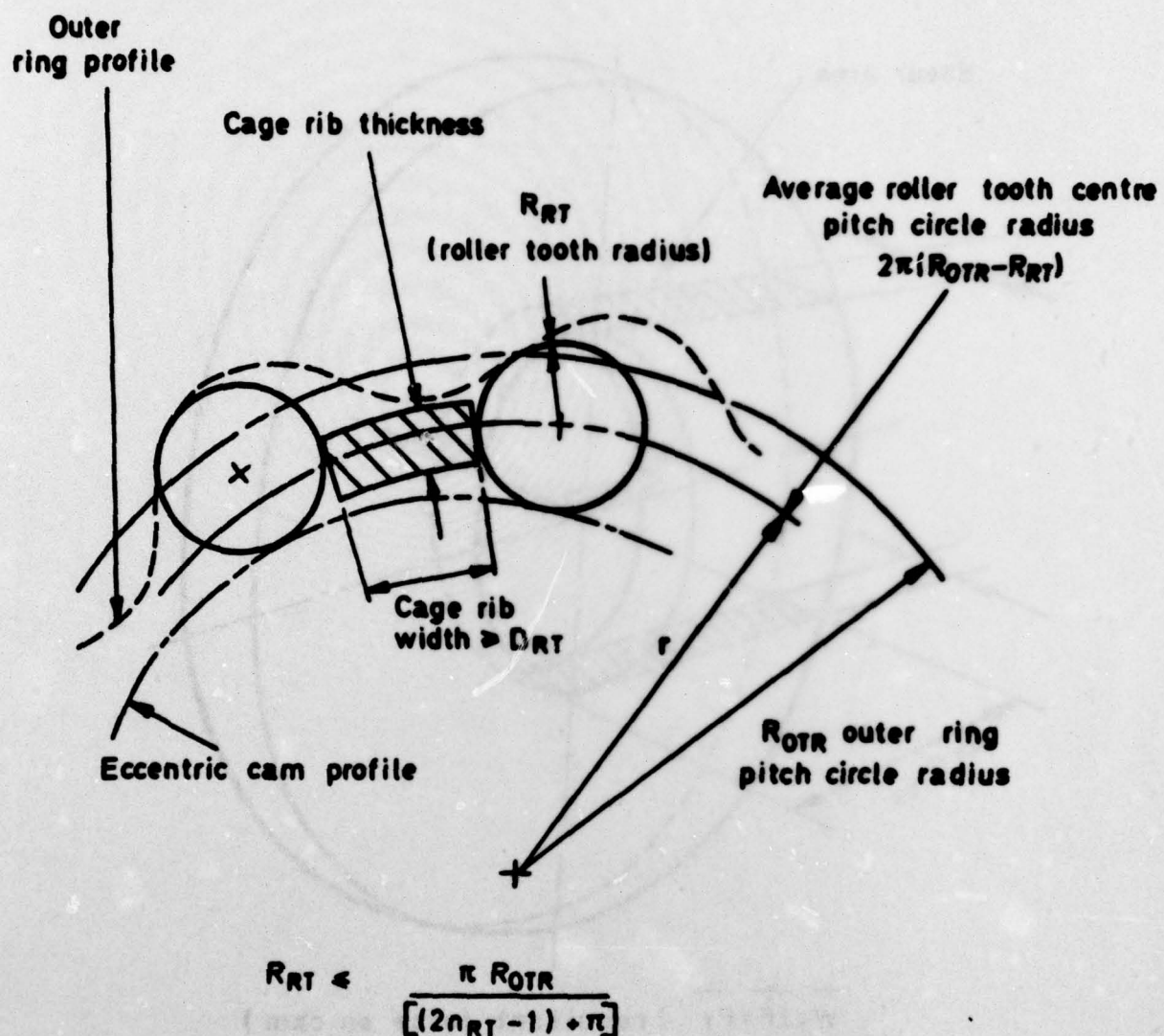Fig 20   Relationship between radius of curvature and gear ratio

Fig 21

$$R_{RT} \leqslant \frac{\pi R_{OTR}}{\left[(2n_{RT}-1)+\pi\right]}$$

Fig 21   Roller tooth radius limitations

Fig 22

$$\overrightarrow{W} = \overrightarrow{IF} + \overrightarrow{P_T} \text{ (resultant force on cam)}$$
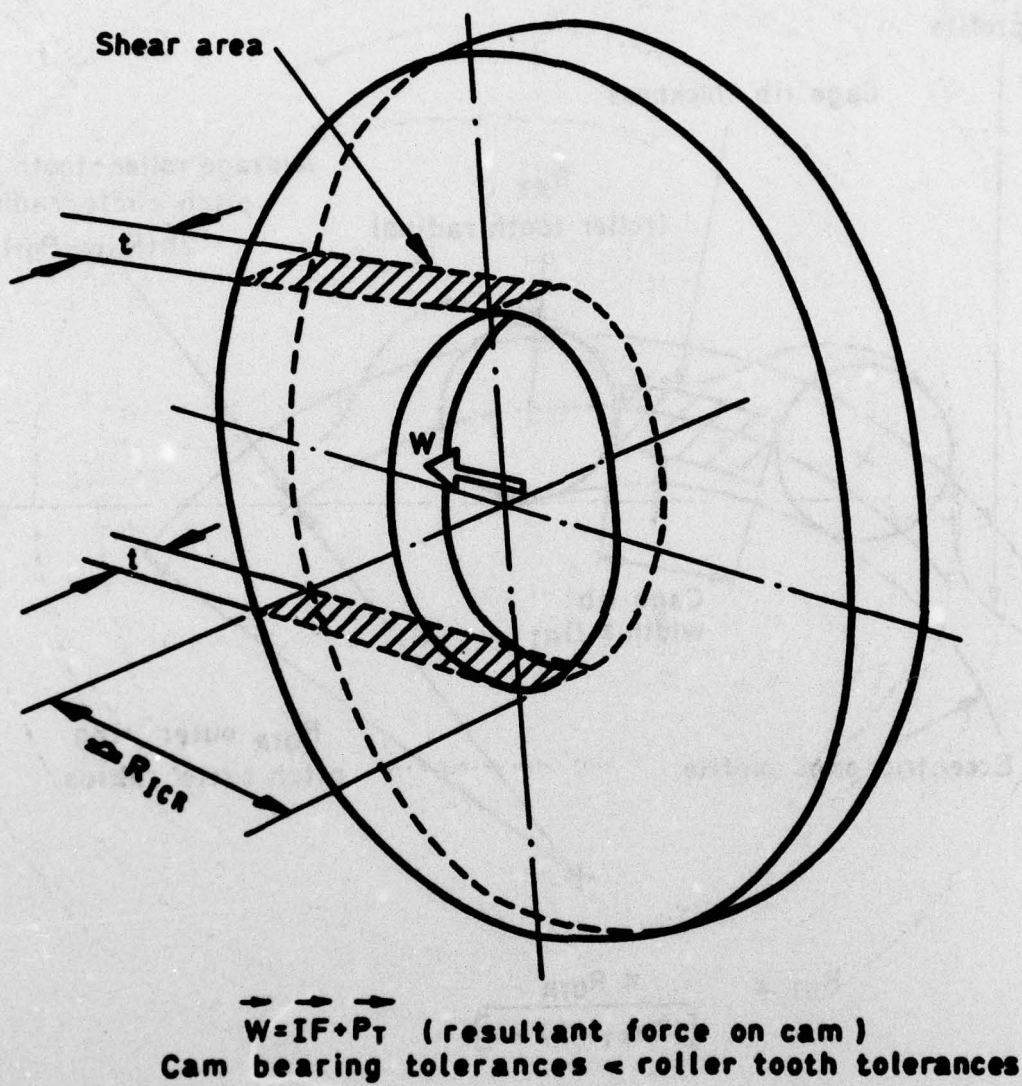Cam bearing tolerances < roller tooth tolerances
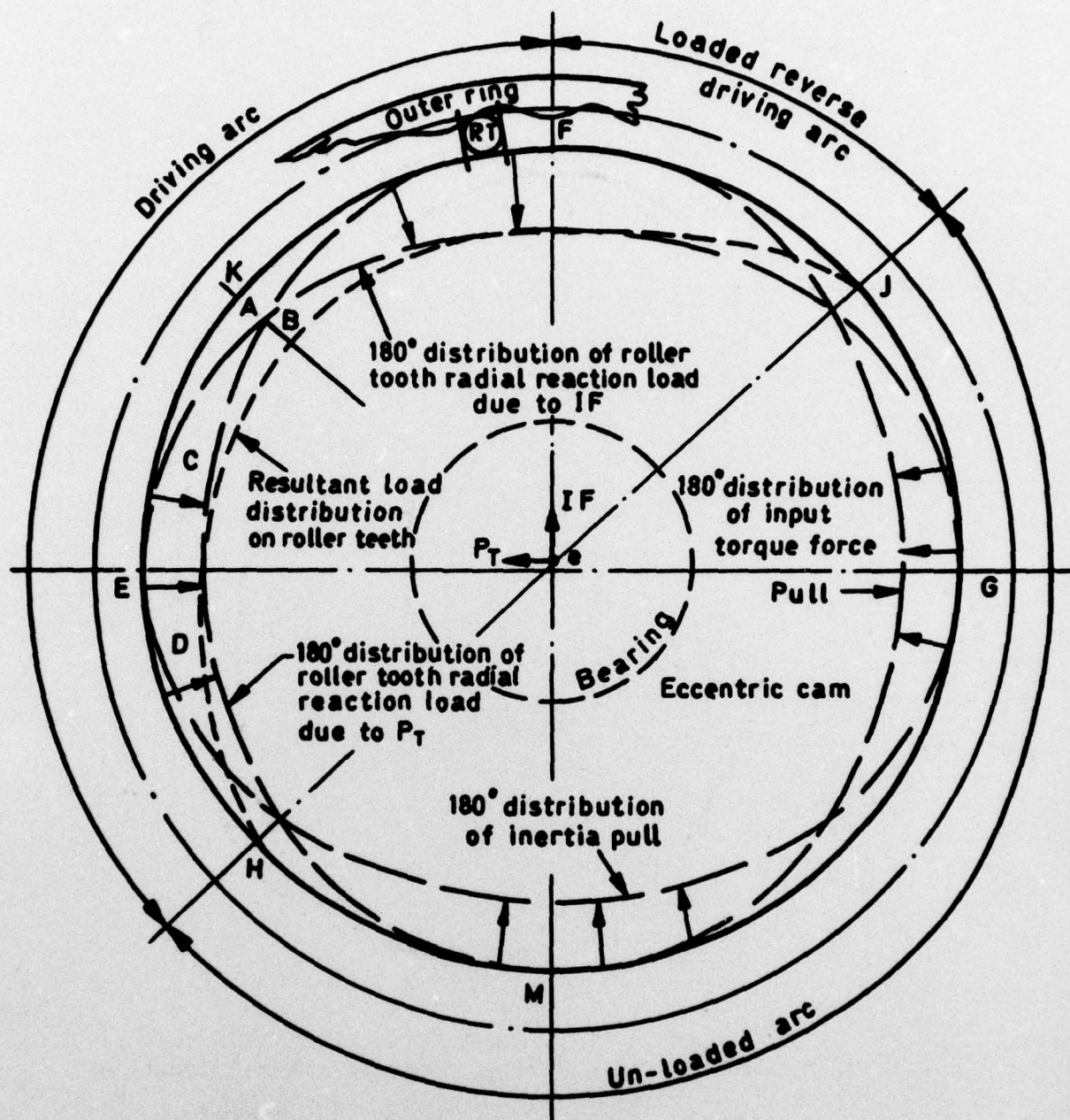
Fig 22   Shear areas in cam

Fig 23

It is considered that the loading distributions vary sinusoidally from zero to maximum to zero in 180°

Fig 23   Suspected load distribution on roller teeth (or outer ring)